Economics Lecture 9

2016-17

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Course Outline

1 Consumer theory and its applications

1.1 Preferences and utility

1.2 Utility maximization and uncompensated demand

1.3 Expenditure minimization and compensated demand

1.4 Price changes and welfare

1.5 Labour supply, taxes and benefits

1.6 Saving and borrowing

2 Firms, costs and profit maximization

2.1 Firms and costs

2.2 Profit maximization and costs for a price taking firm

3. Industrial organization

3.1 Perfect competition and monopoly

3.2 Oligopoly and games

2.2 Profit maximization and costs for a price taking firm

2.2 Profit maximization and costs for a price taking firm

1. Definition of price taking
2. The shutdown and output rules
3. Profit maximization by a price taking CRS firm
4. Cost curves, profit maximization and supply with decreasing returns to scale
5. Cost curves with increasing returns to scale
6. Cost curves and supply with a u-shaped average cost curve
7. Long run and short run costs and supply

1. Definition of price taking

A firm is a price taker if nothing it can do affects the prices it pays for inputs and outputs.

In particular its output quantity does not affect its output price.

Price taking does not mean prices do not change, just they do not change because of the firm will.

Price taking is plausible if the firm has a small market share.

2. The shutdown & output rules

2. The shutdown & output rules

The shutdown rule and average cost.

$c(v, w, q)$ is total cost

so $\dfrac{c(v, w, q)}{q}$ is average cost, AC

revenue pq, total cost $c(v, w, q)$

$$\text{profits} = pq \quad - \quad c(v, w, q)$$

Why we need to know about average costs.

The shutdown rule

If the cost of producing 0 is 0, i.e. $c(v, w, 0) = 0$
the firm can make 0 profits by
producing 0 so if the firm maximizes profits at $q > 0$
profits $= pq - c(v, w, q) \geq 0$ so

$$\mathrm{p} \geq \frac{c(v, w, q)}{q} = \mathrm{AC}$$

Why we need to know about average costs.

The shutdown rule

The firm shuts down (does not produce) if price < AC at all levels of output.

The shut down rule implies that if a profit maximising firm produces $q > 0$ then price $\geq$ AC.

The output rule and marginal cost

$$\text{Profit} = \text{revenue} - \text{cost} = \text{R(q)} - \text{c(v, w, q)}$$

$$\text{If marginal revenue} = \frac{\partial \text{R}}{\partial q} > \frac{\partial \text{c}}{\partial q} = \text{marginal cost}$$

then increasing output q increases profits.

$$\text{If marginal revenue} = \frac{\partial \text{R}}{\partial q} < \frac{\partial \text{c}}{\partial q} = \text{marginal cost}$$

then increasing output q decreases profits.

The output rule and marginal cost

Why we need to know about marginal costs.

Profit maximization at $q > 0$ implies

$$\text{marginal revenue} = \frac{\partial R}{\partial q} = \frac{\partial c}{\partial q} = \text{marginal cost}.$$

But marginal revenue = marginal cost

does not necessarily imply profit maximization.

You will see an example of this shortly.

What is marginal revenue?

- By definition for a price taking firm

marginal revenue = price

and does not vary with output

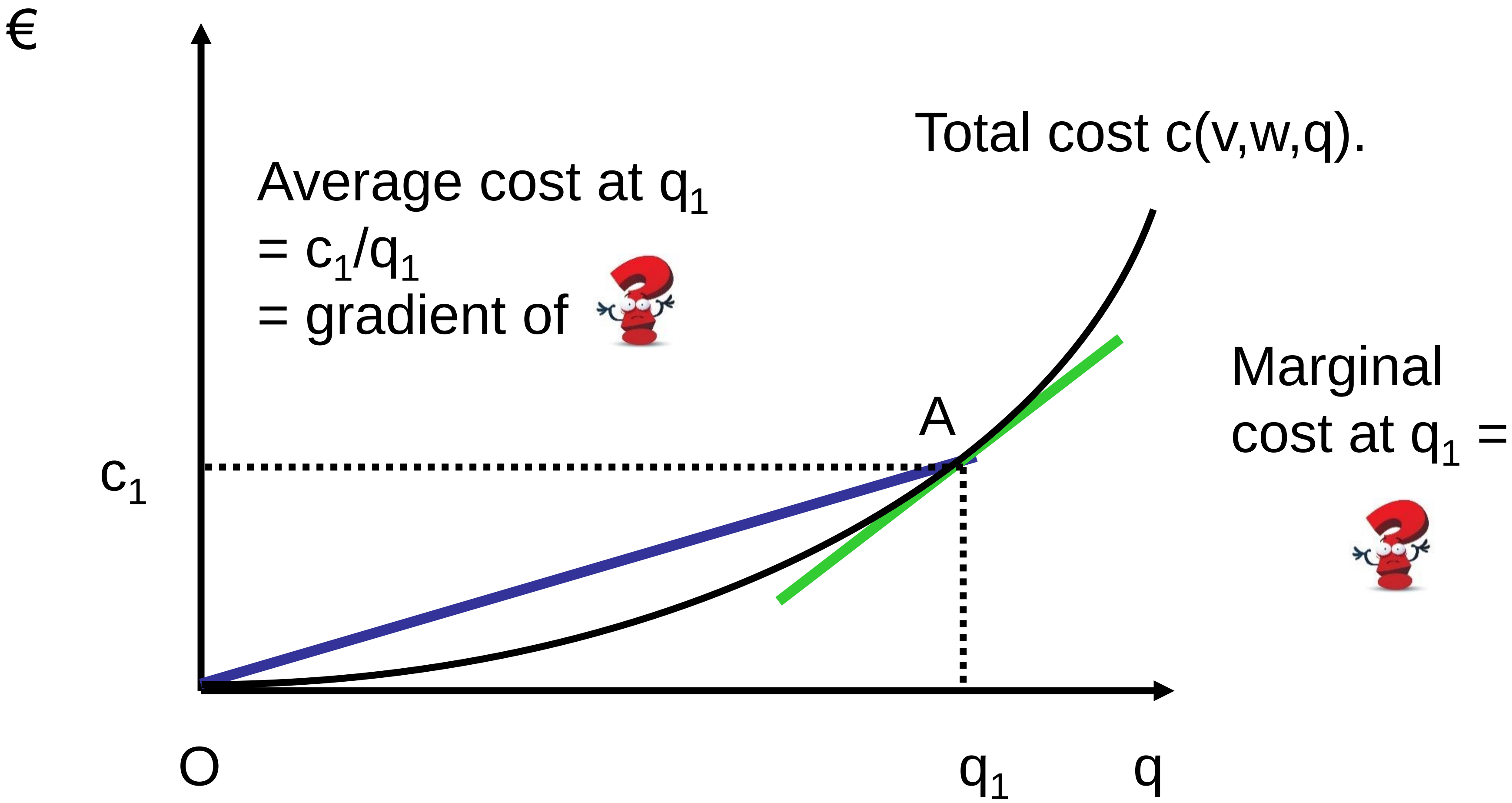
(note that perfect competition implies price taking)

- For a monopoly marginal revenue depends on the firm’s output

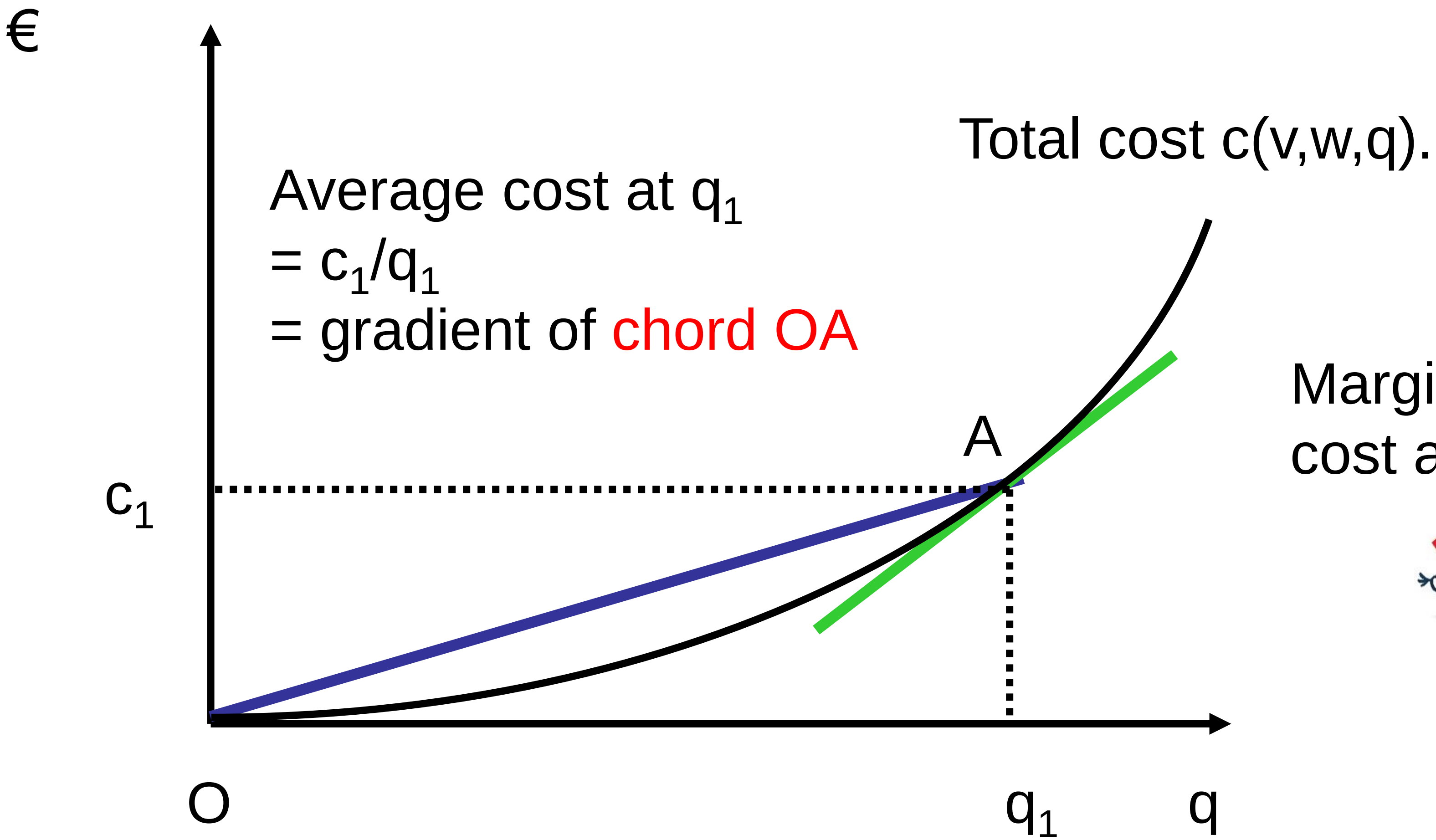
- For a firm in an oligopoly marginal revenue depends on the firm’s output & the output of other firms.

The relationship between marginal cost & average cost

Total marginal and average costs

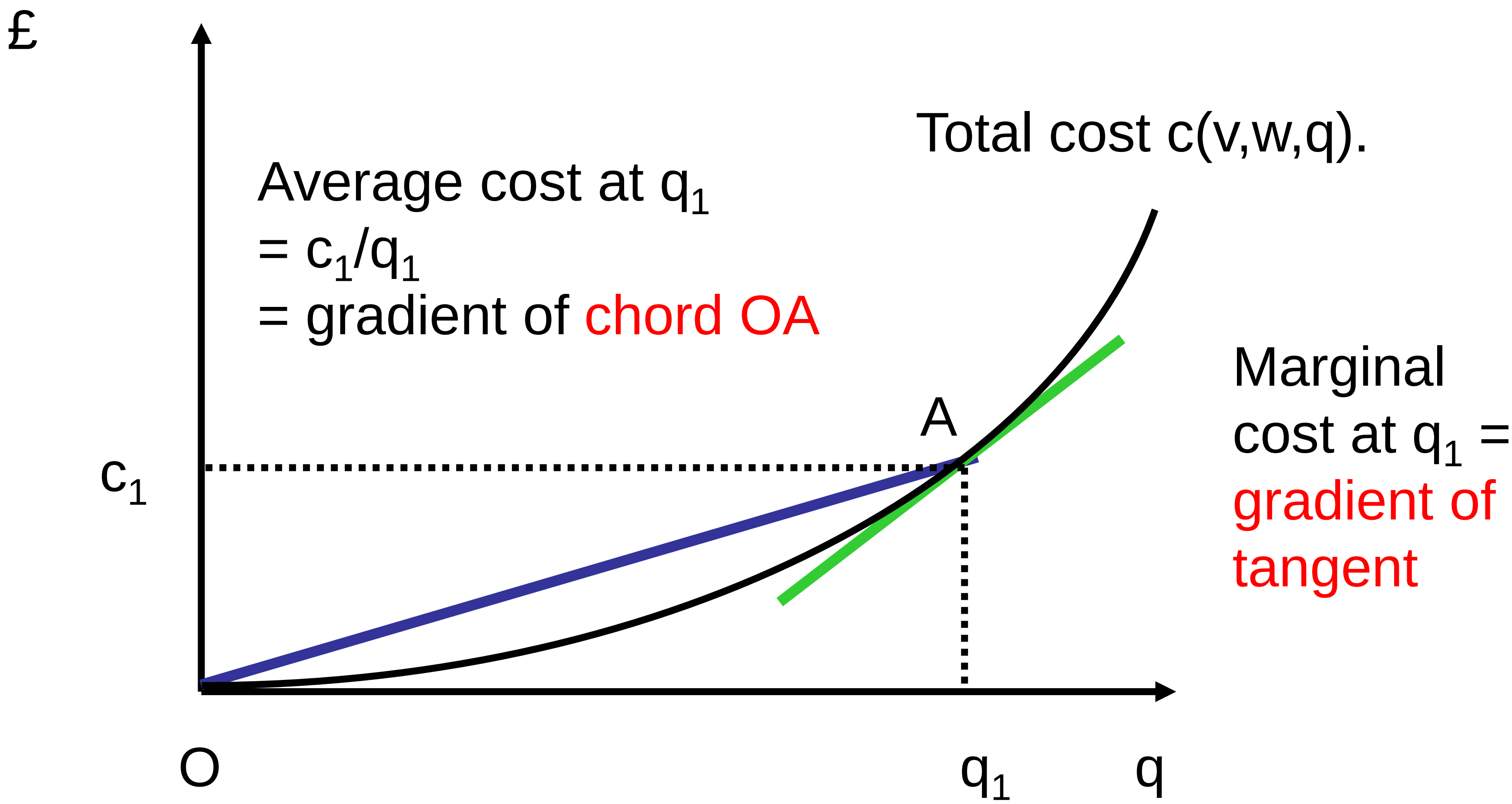


Total marginal and average costs



Marginal cost at q_1 =

Total marginal and average costs



So average cost increases when

MC  AC

average cost decreases when

MC  AC

When MC  AC the AC function has a critical point (maximum, minimum or point of inflection.)

So average cost increases when

MC $>$ AC

average cost decreases when

MC  AC

When MC  AC the AC function has a critical point (maximum, minimum or point of inflection.)

So average cost increases when

MC > AC

average cost decreases when

MC < AC

When MC  AC the AC function has a critical point (maximum, minimum or point of inflection.)

So average cost increases when

MC $>$ AC

average cost decreases when

MC $<$ AC

When MC $=$ AC the AC function has a critical point (maximum, minimum or point of inflection.)

3. Profit maximization by a price taking for a constant returns to scale (CRS) firm

3. Profit maximization by a price taking CRS firm

Cost curves with CRS

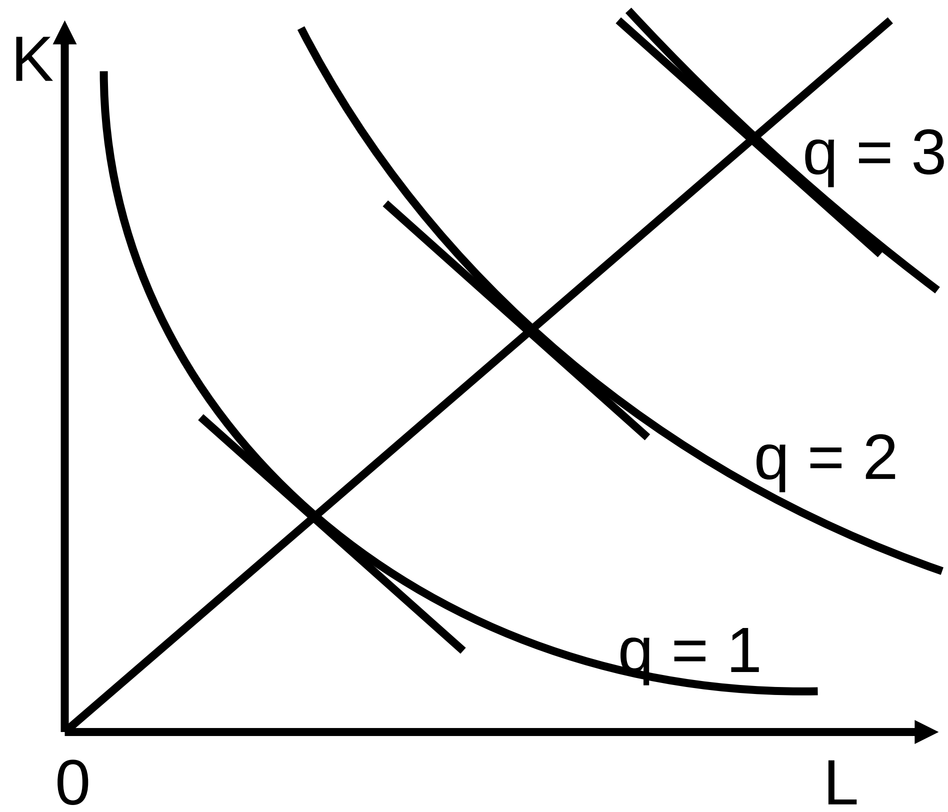
The production function

q = f(K,L) has constant returns

to scale if

for all positive numbers m

mf(L,K) = f(mL,mK).



isoquants with CRS

Under **constant returns to scale** CRS the optimal ratio of inputs (e.g. the capital labour ratio)

is the same at all levels of output.

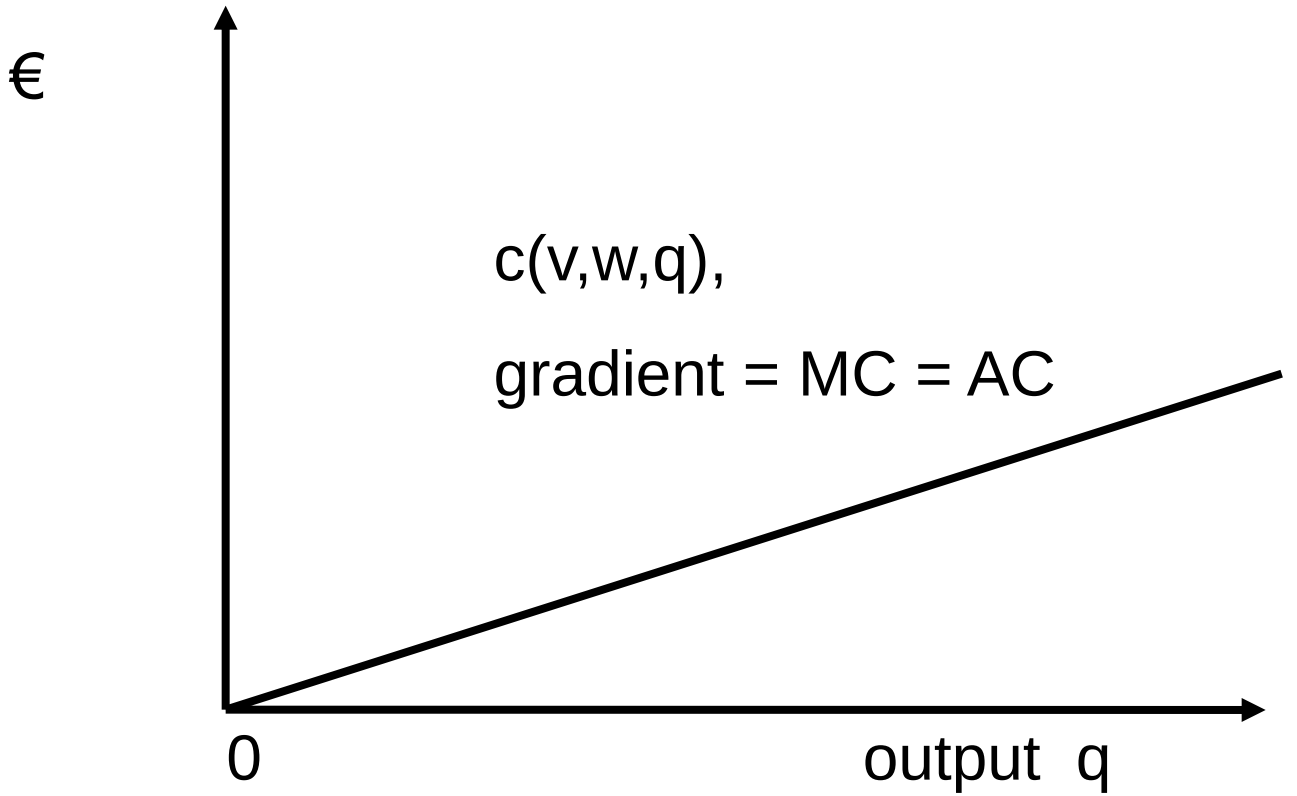
Multiplying inputs by 2 multiplies output by 2.

So it costs twice as much to produce 2 units of output as it costs to produce 1 unit of output.

More generally with constant returns to scale, given input prices v and w

$c(v,w,mq) = mc(v,w,q)$.

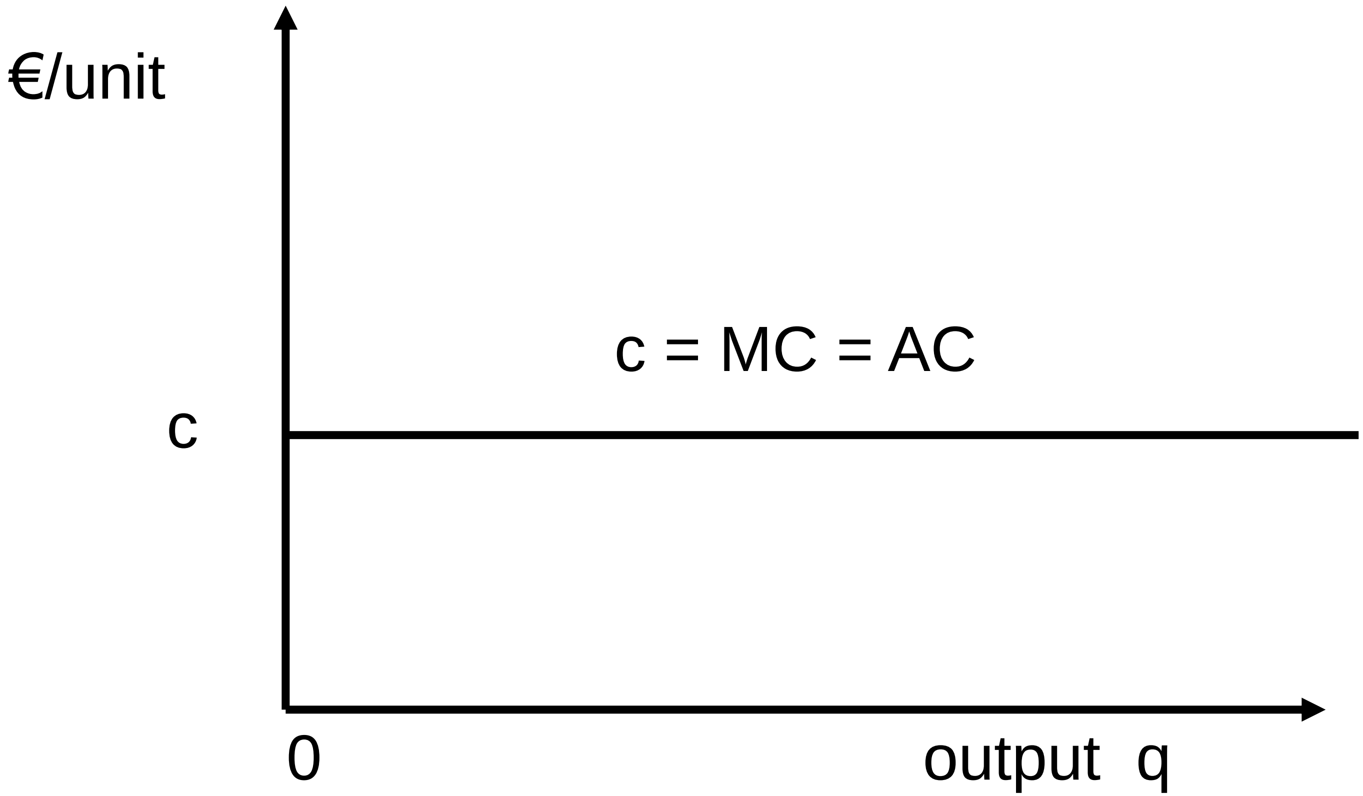
Total cost function from a CRS production function



Marginal cost = average cost

vary with input prices v and w but not with output

Marginal and average cost from a CRS production function



Marginal cost = average cost =c

varies with input prices v and w but not with output

- A firm is a price taker if nothing it can do changes the price p at which it sells.
- Profits = $pq – cq = (p – c)q$
- If $p > c$, so $(p – c) > 0$ increasing q always increases profits, there is no profit maximizing output.
- If $p < c$ so $(p – c) < 0$ the firm makes losses at all $q > 0$ so produces 0.
- If $p = c$ the firm makes 0 profit at any q.

$p = c$ is the only price at which a price taking firm with constant returns to scale has a profit maximum at $q > 0$.

4. Cost curves profit maximization & supply with decreasing returns to scale

4. Cost curves, profit maximization and supply with decreasing returns to scale

Under <u>**decreasing returns to scale**</u> DRS multiplying inputs by 2 multiplies output by  than 2.

So it costs  than twice as much to produce 2 units of output as it costs to produce 1 unit of output, so

c(v,w,2q)   2c(v,w,q).

4. Cost curves, profit maximization and supply with decreasing returns to scale

Under **<u>decreasing returns to scale</u>** DRS multiplying inputs by 2 multiplies output by less than 2.

So it costs more than twice as much to produce 2 units of output as it costs to produce 1 unit of output, so

$c(v,w,2q)$  $2c(v,w,q)$.

4. Cost curves, profit maximization and supply with decreasing returns to scale

Under **decreasing returns to scale** DRS multiplying inputs by 2 multiplies output by less than 2.

So it costs more than twice as much to produce 2 units of output as it costs to produce 1 unit of output, so

$c(v,w,2q) > 2c(v,w,q)$.

Cost curves with DRS

More generally with decreasing returns to scale, given input prices if **m > 1**

$c(v,w,mq) \quad > \quad m\, c(v,w,q)$

so AC at output mq = $\dfrac{c(v,w,mq)}{mq}$  $\dfrac{c(v,w,q)}{q}$ = AC output q

AC  with output.

Cost curves with DRS

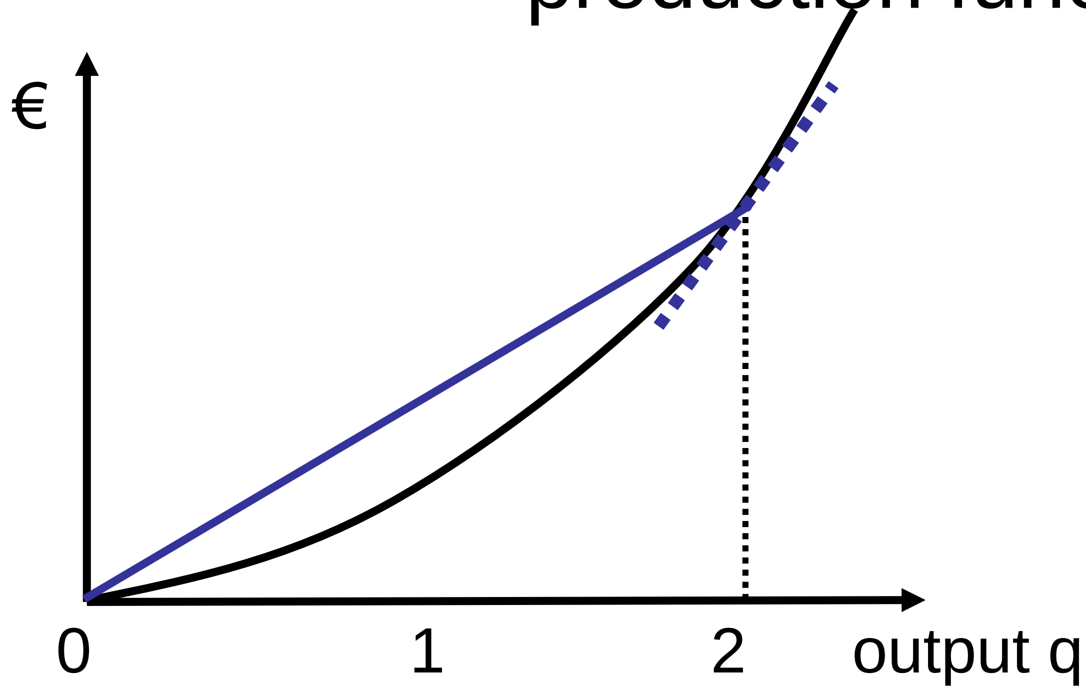
More generally with decreasing returns to scale, given input prices if **m > 1**

$c(v,w,mq) \quad > \quad m\, c(v,w,q)$

so AC at output mq = $\dfrac{c(v,w,mq)}{mq} \quad > \quad \dfrac{c(v,w,q)}{q}$ = AC output q

AC increases with output.

Total cost function from a DRS production function



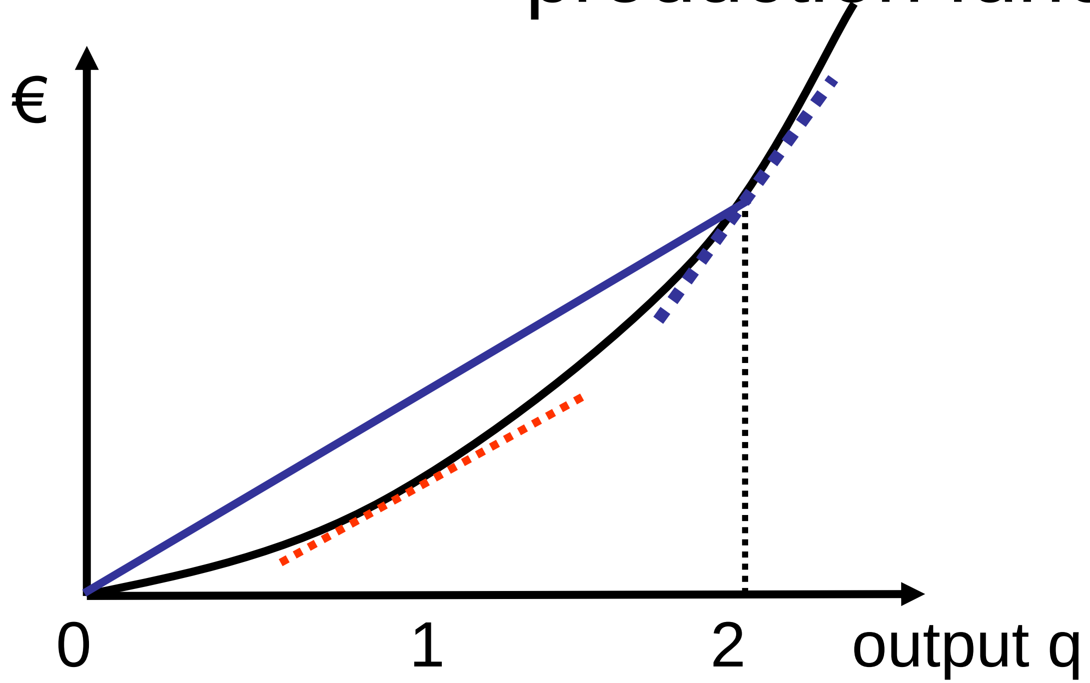
MC = gradient of tangent   AC = gradient of chord.

MC  as q increases.

AC  as q increases.

MC  AC.

Total cost function from a DRS production function



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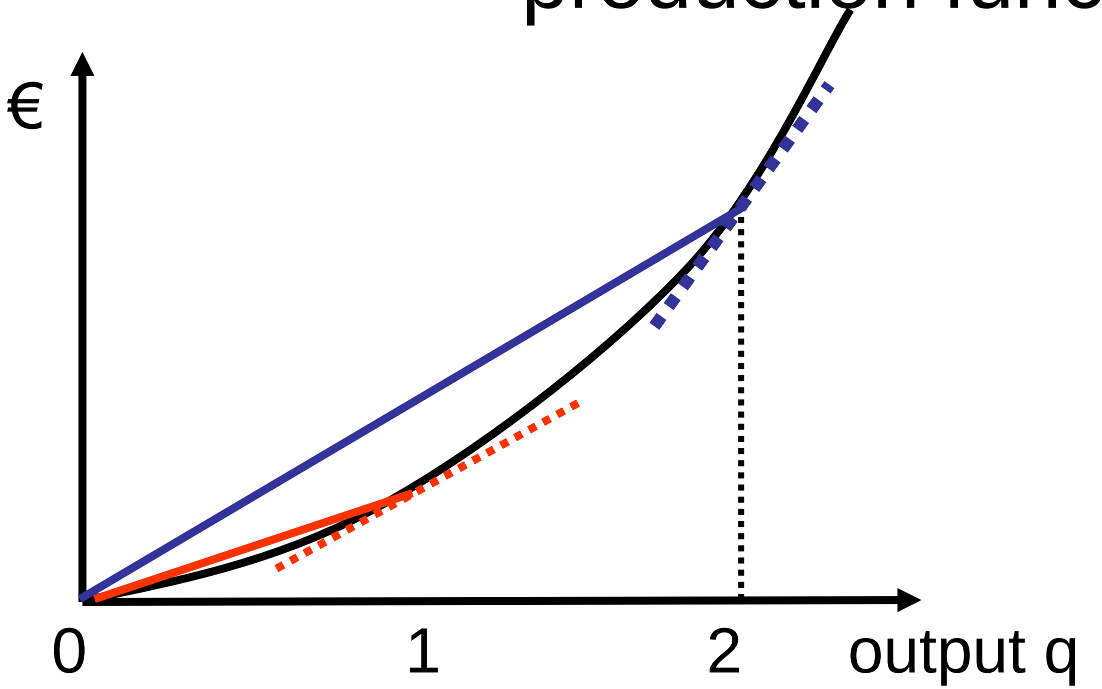
Total cost function from a DRS production function



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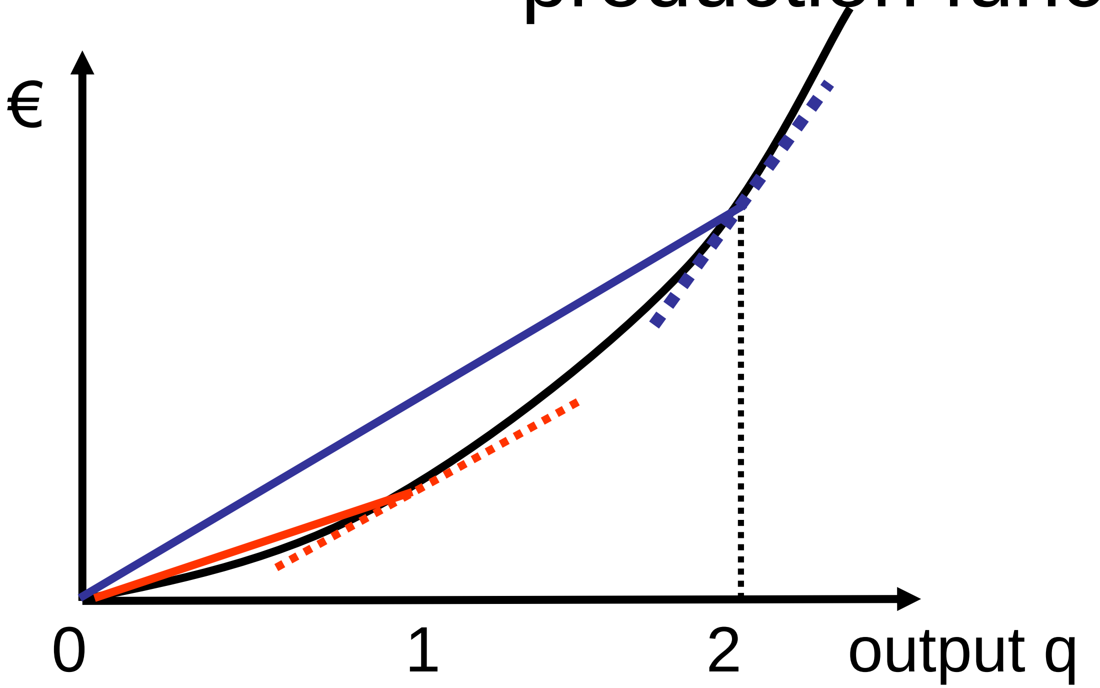
Total cost function from a DRS production function



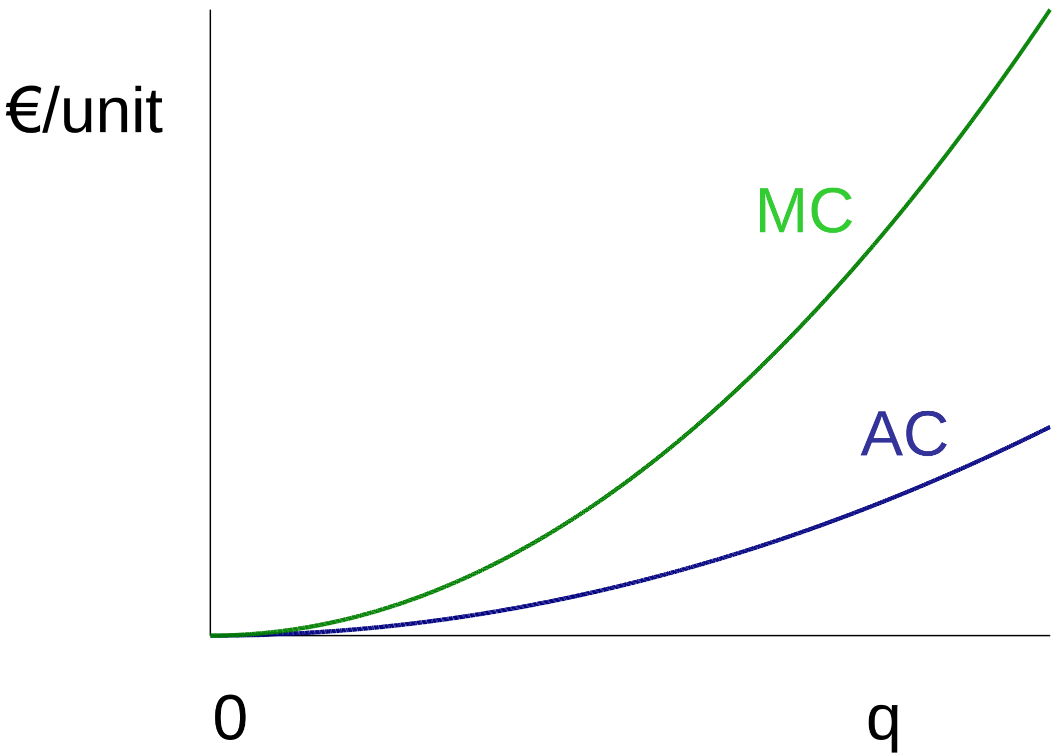
MC = gradient of tangent   AC = gradient of chord.

MC increases as q increases.

AC increases as q increases.

MC > AC.

AC and MC from a DRS production function.



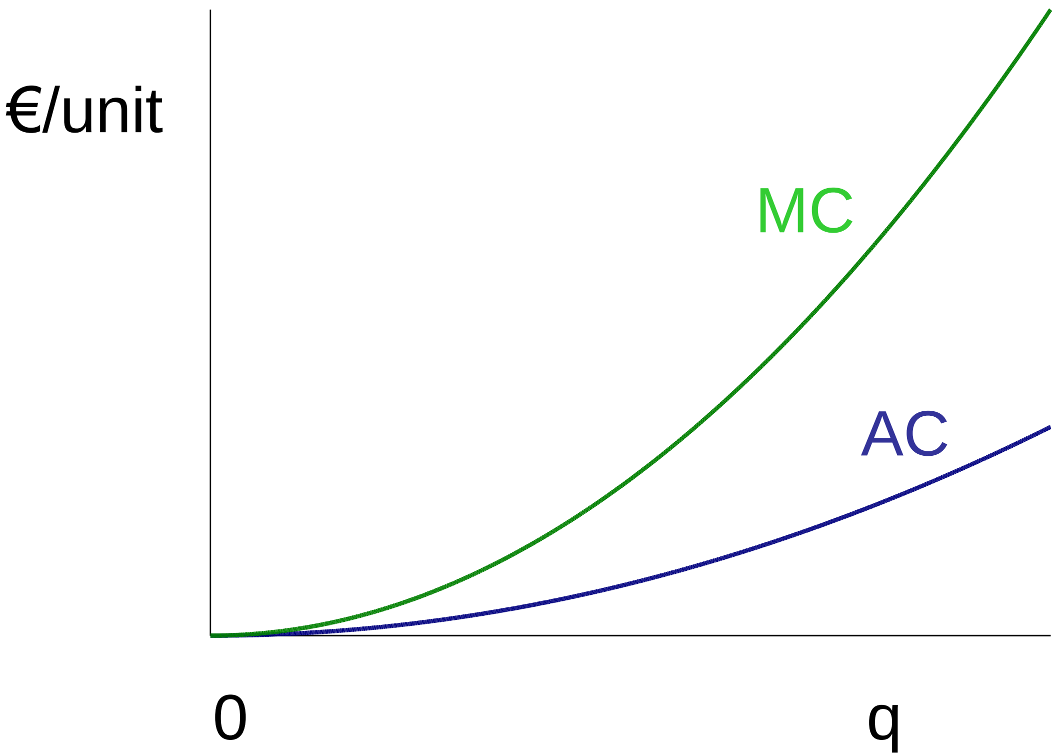
MC  AC
everywhere

MC 

as q increases

AC 
as q increases

AC and MC from a DRS production function.



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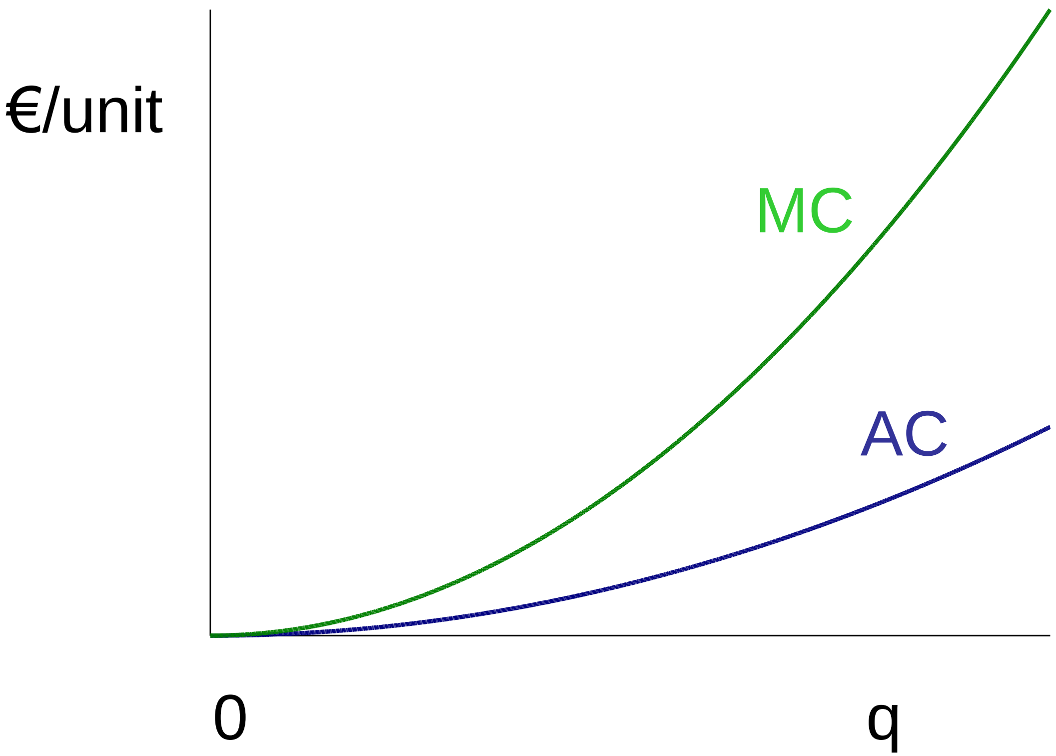

MC > AC everywhere

MC increases

as q increases

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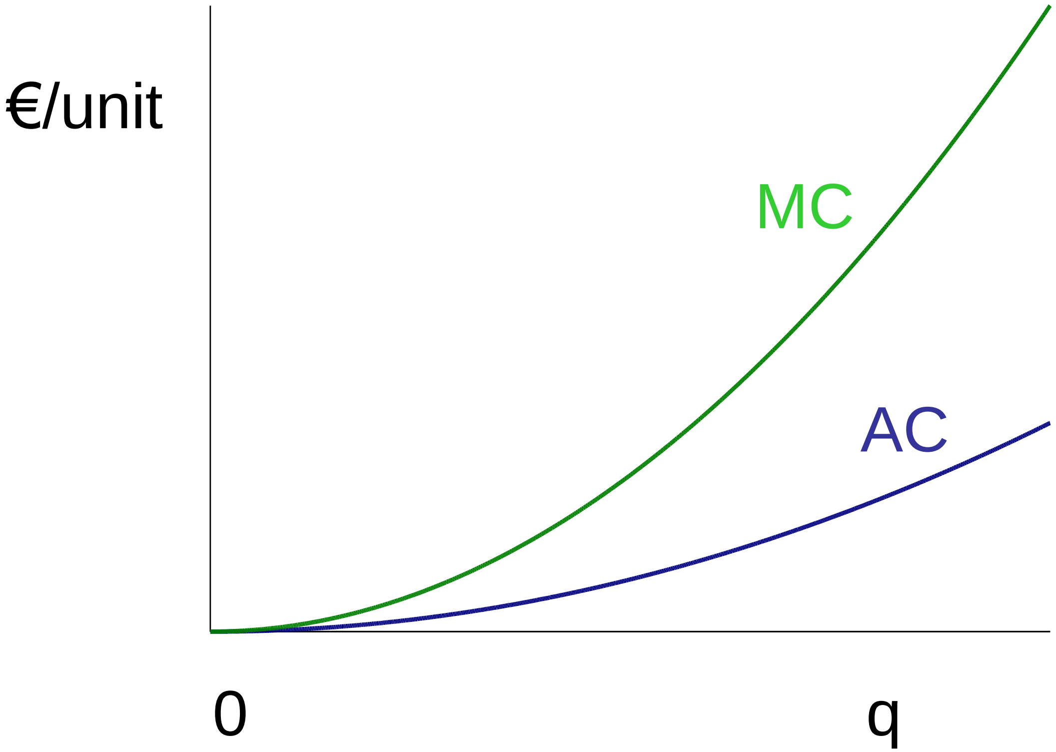
AC and MC from a DRS production function.



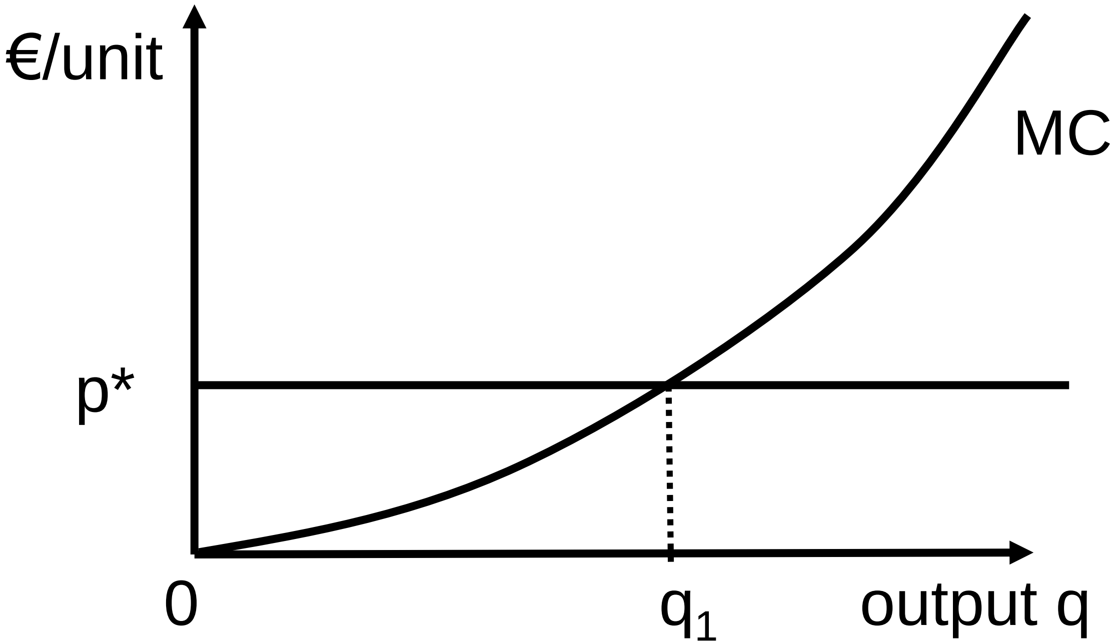
MC > AC everywhere

MC increases

as q increases

AC increases as q increases

If MC is increasing p = MC gives a profit maximum: intuition



When $q < q_1$ $MC < p^*$, increasing output by 1 unit increases costs by MC and revenue by p*. As $p^* > MC$ this increases profits.

When $q > q_1$ $MC > p^*$, increasing output by 1 unit increases costs by MC and revenue by p*. As $p^* < MC$ this decreases profits.

If MC is increasing p = MC gives a profit maximum: calculus

$\pi(q) = pq - c(q)$ ($c(q)$ = total cost)

First order condition for profit maximization

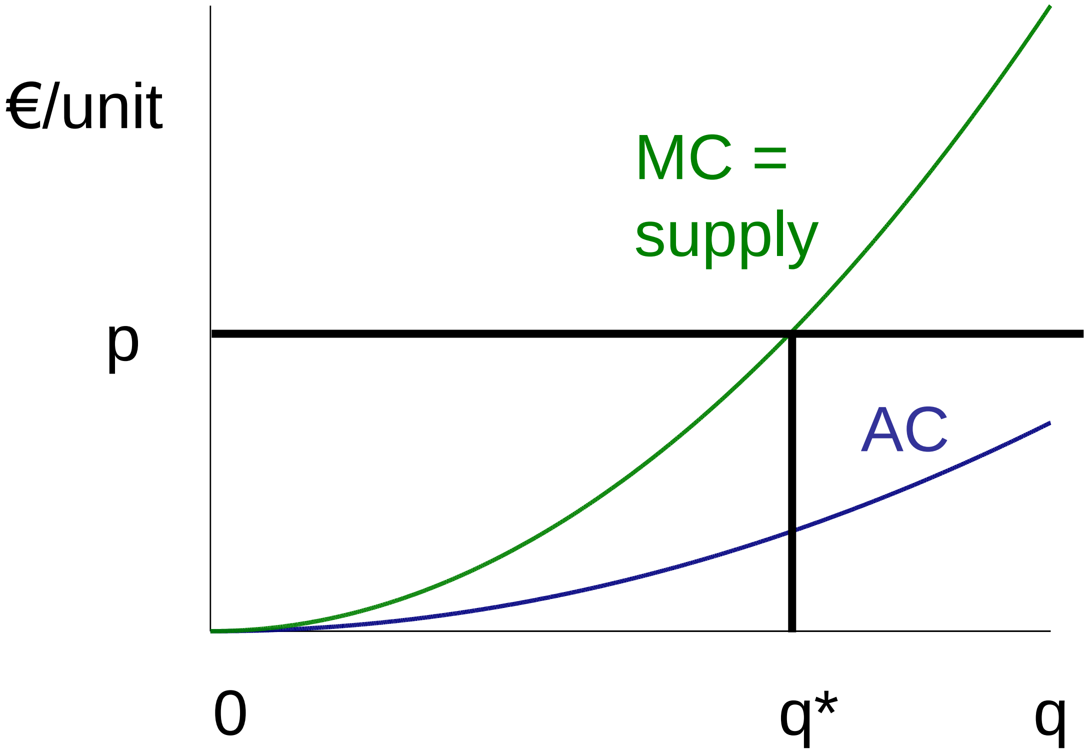
$\pi'(q) = p - c'(q)$ = price – marginal cost = 0.

As $c''(q)$ is the derivative of $c'(q)$ = MC increasing marginal cost implies that $c''(q) > 0$

The second derivative $\pi''(q) = - c''(q) < 0$ so $\pi(q) = pq - c(q)$ is a concave function of q.

The first order conditions give a maximum.

Profit maximization with price taking and a DRS production function



With DRS MC is increasing and MC > AC

If p = MC then p > AC so the shutdown rule is satisfied.

The MC curve is the supply curve. The firm makes profits.

5. Cost curves profit maximization & supply with increasing returns to scale

5. Cost curves with IRS

Under <u>increasing **returns to scale**</u> IRS multiplying inputs by 2 multiplies output by <u>**more**</u> than 2.

So it costs <u>**less**</u> than twice as much to produce 2 units of output as it costs to produce 1 unit of output, so

$c(v,w,2q) < 2c(v,w,q)$.

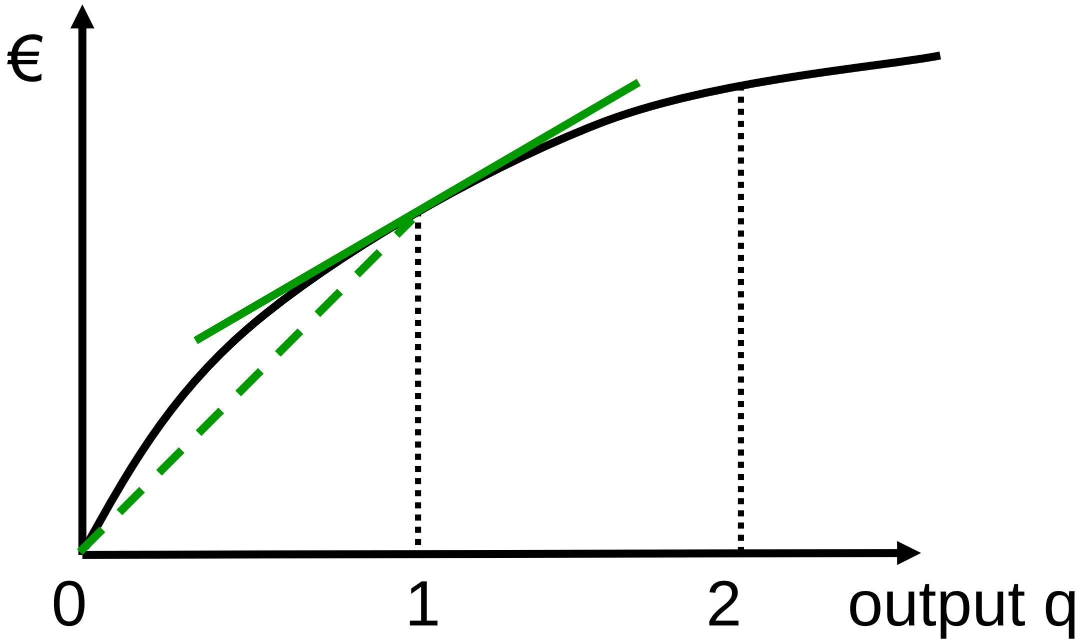
More generally with increasing returns to scale, given input prices if $m > 1$

$c(v,w,mq) < m\, c(v,w,q)$

so AC at output mq $= \dfrac{c(v,w,mq)}{mq} < \dfrac{c(v,w,q)}{q} =$ AC at output q

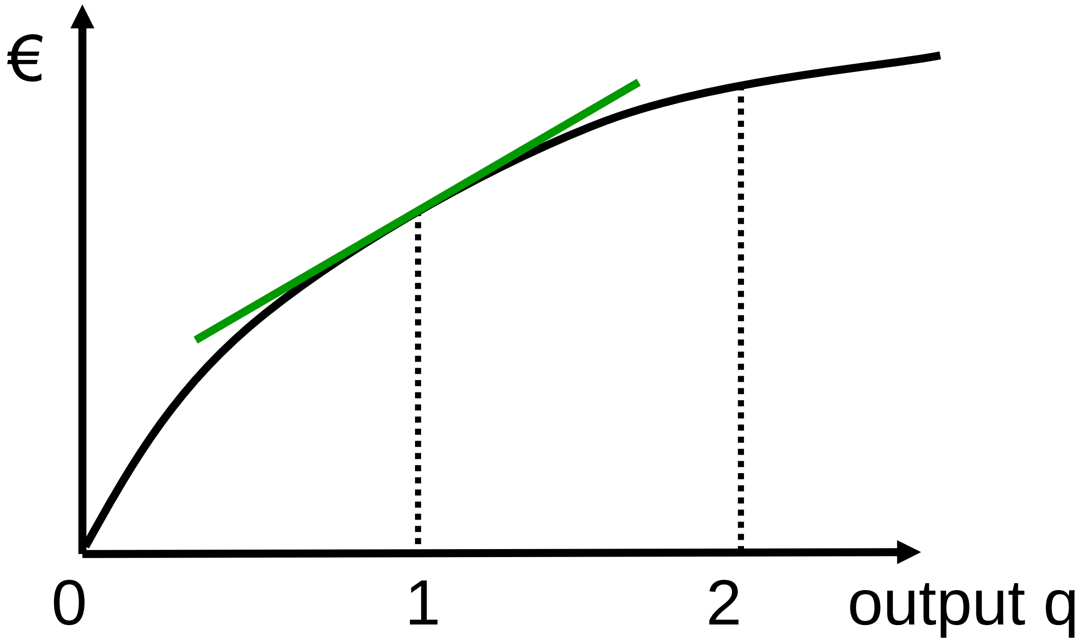
AC <u>**decreases**</u> with output.

Total cost function from an IRS production function



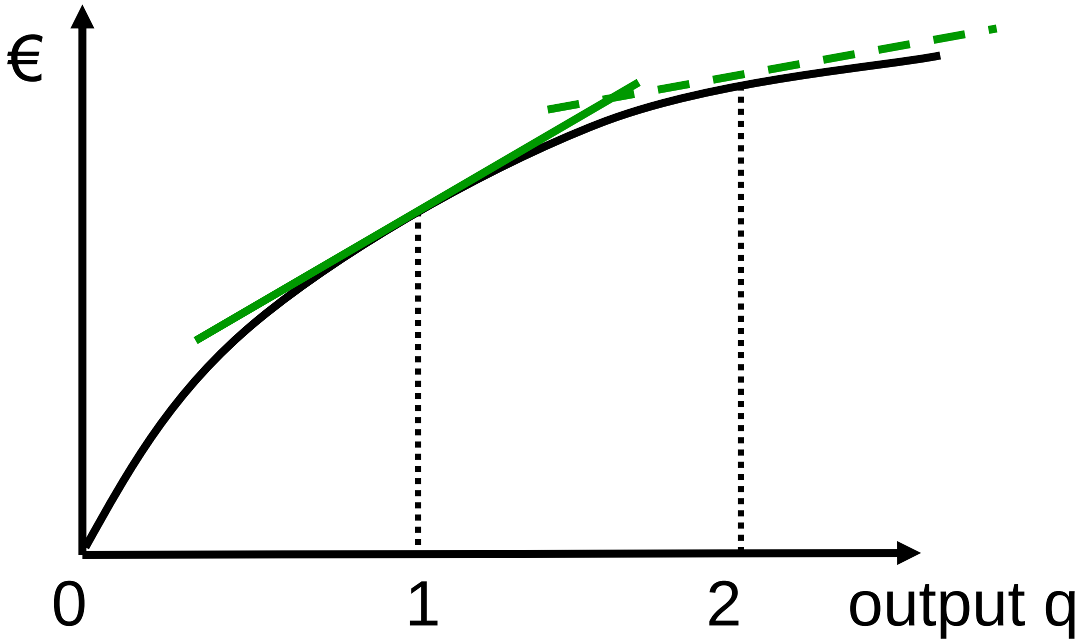
MC = gradient of tangent < AC = gradient of chord

Total cost function from an IRS production function



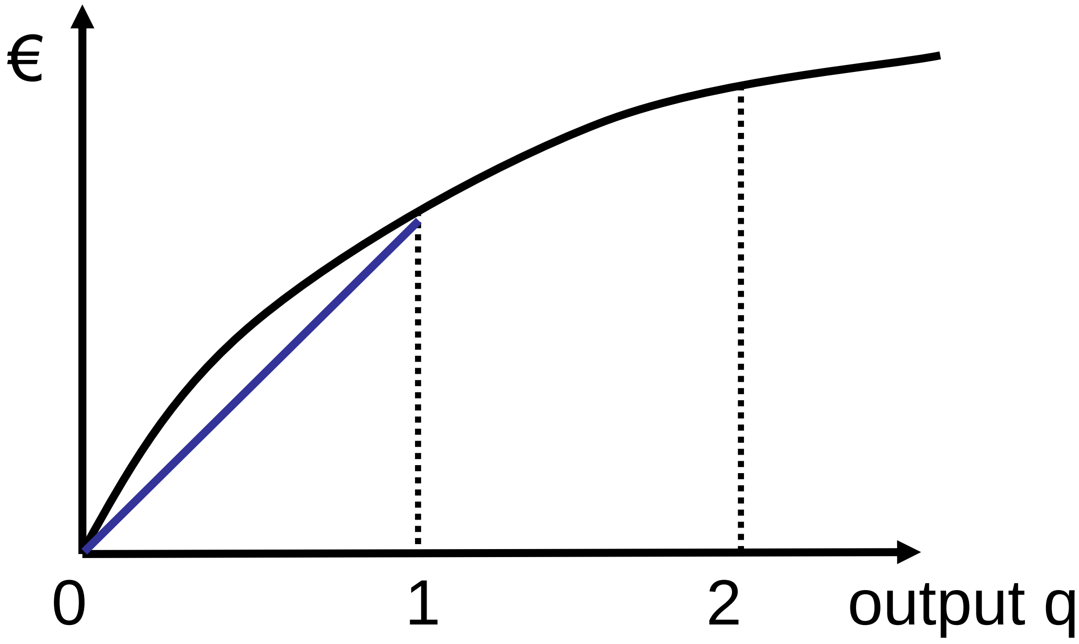
MC decreases as q increases

Total cost function from an IRS production function



MC decreases as q increases

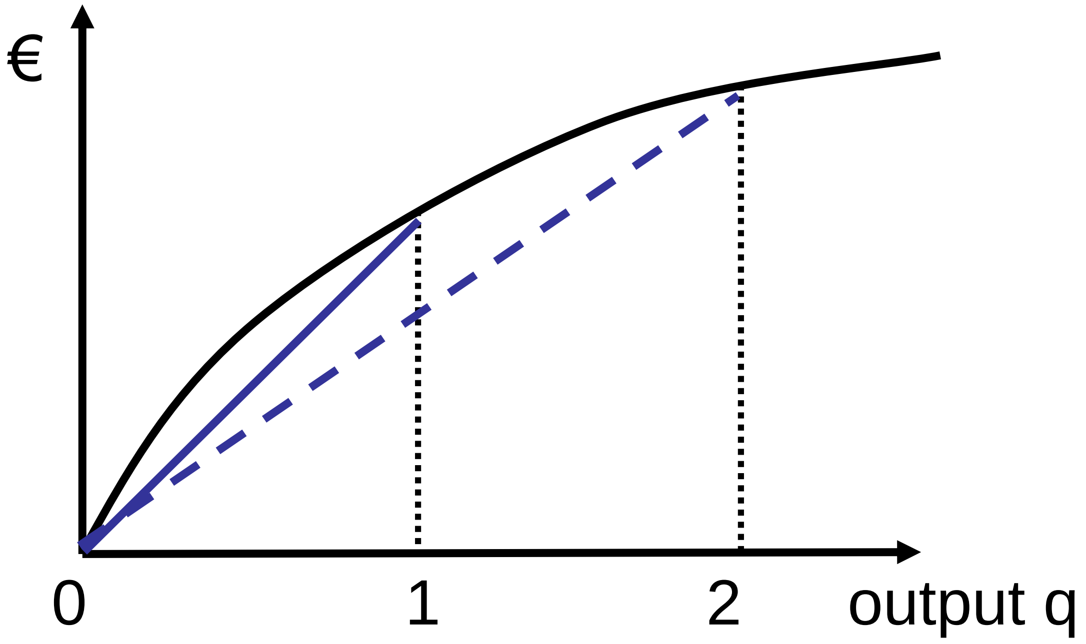
Total cost function from an IRS production function



AC decreases as q increases,

there are **economies of scale**

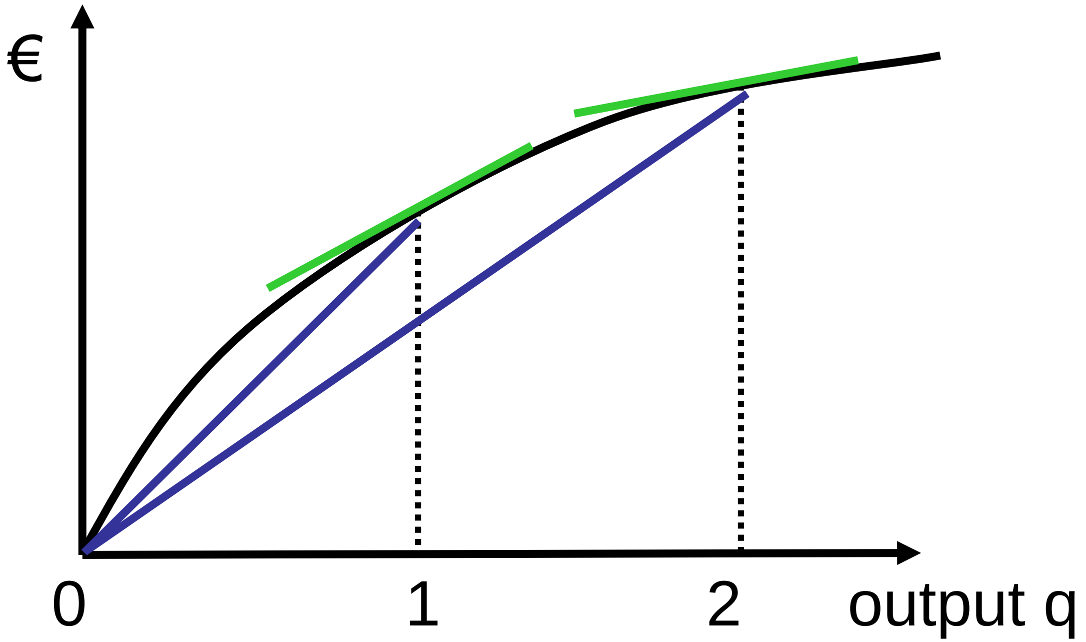
Total cost function from an IRS production function



AC decreases as q increases,

there are **economies of scale**

Total cost function from an IRS production function



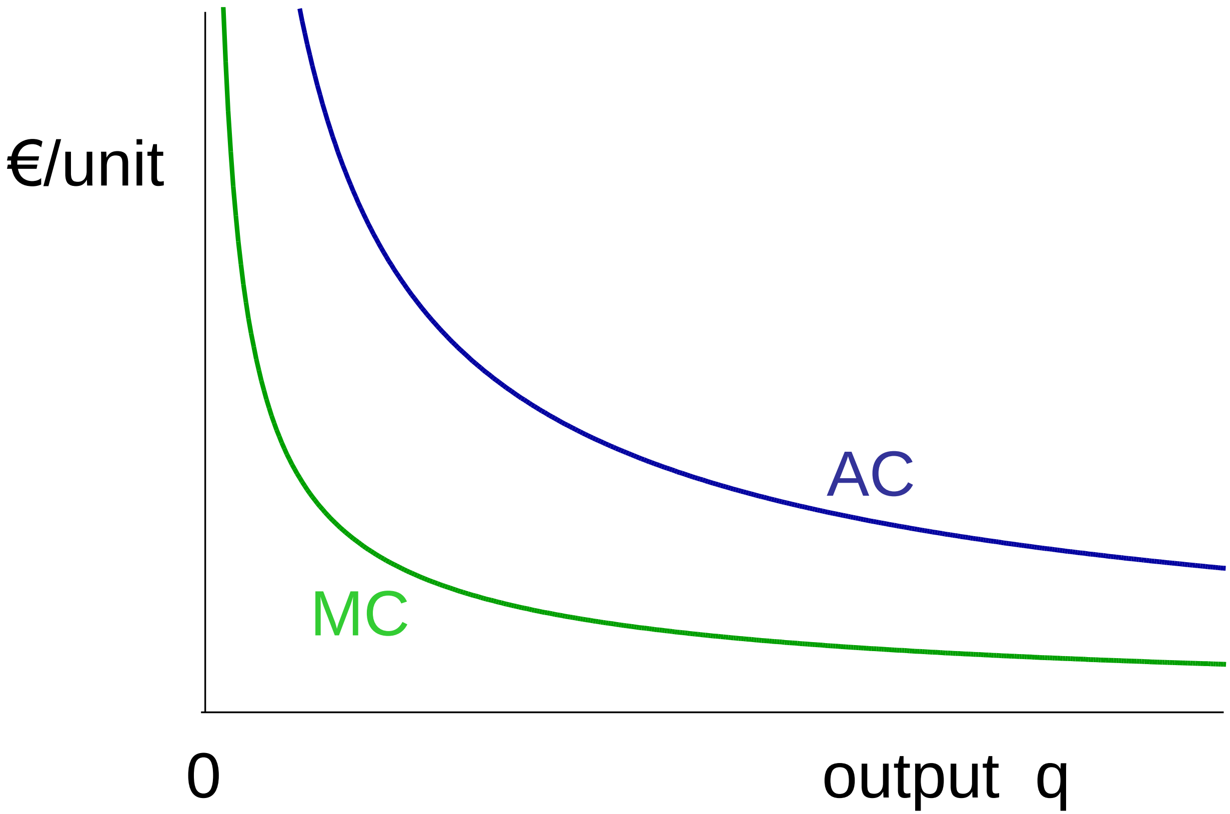
MC = gradient of tangent < AC = gradient of chord

MC decreases as q increases

AC decreases as q increases,

there are **economies of scale**

AC and MC from an IRS production function



MC < AC everywhere

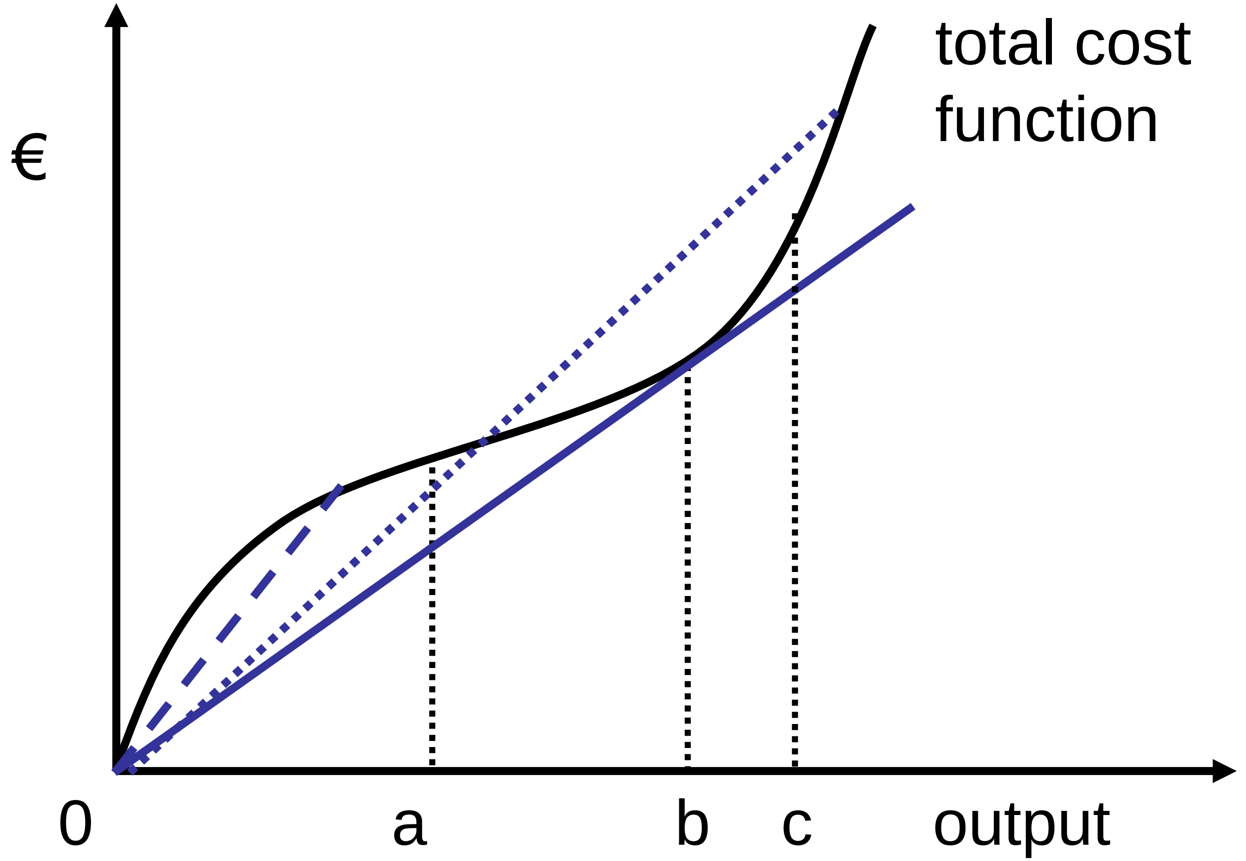
MC ↓ (decreases) as q ↑

AC ↓ (decreases) as q ↑

6. Cost curves and supply with a u-shaped average cost curve

6. Cost curves and supply with a u-shaped average cost curve

- Often assumed for perfect competition.
- For small q and there are economies of scale → AC falls.
- For large q and there are diseconomies of scale → AC rises.

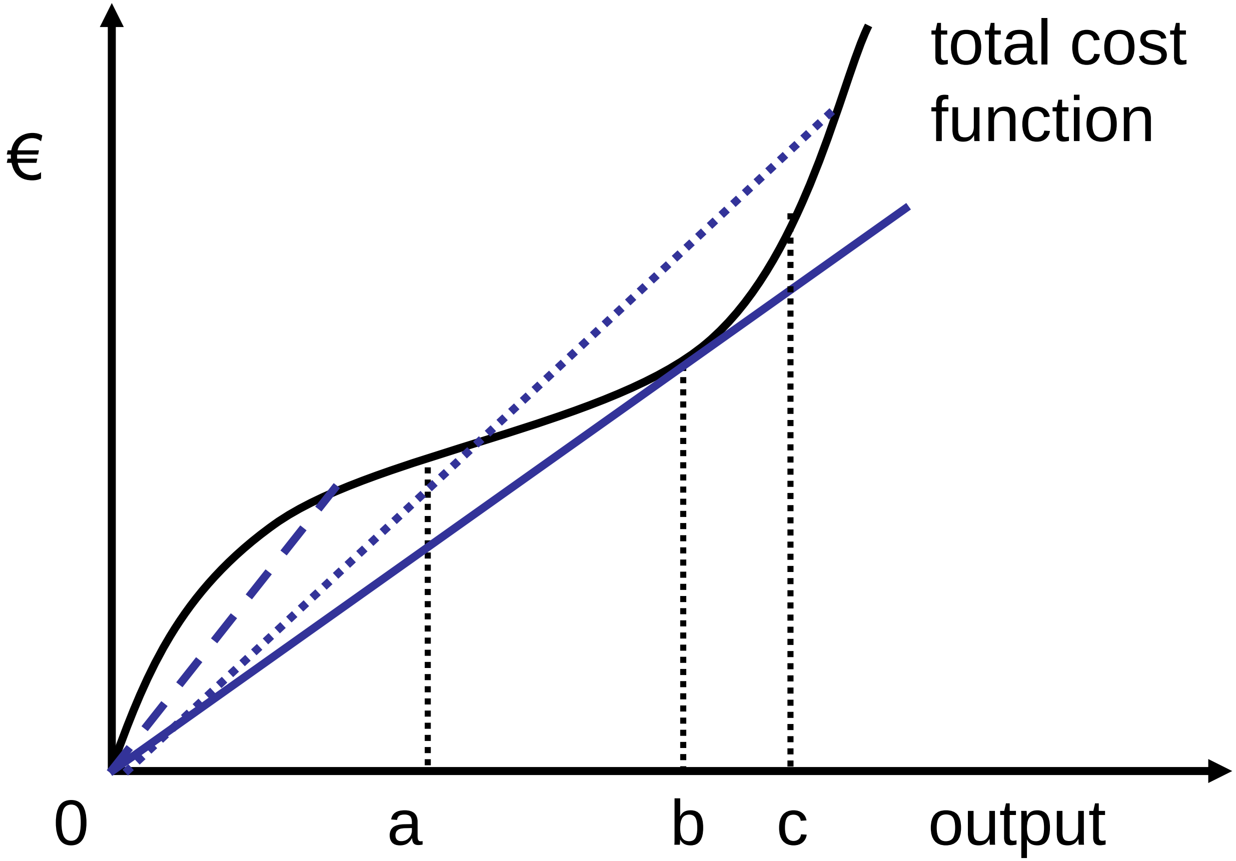


AC falls when 

AC rises when

AC has a minimum at

At the minimum AC = MC.   The chord is

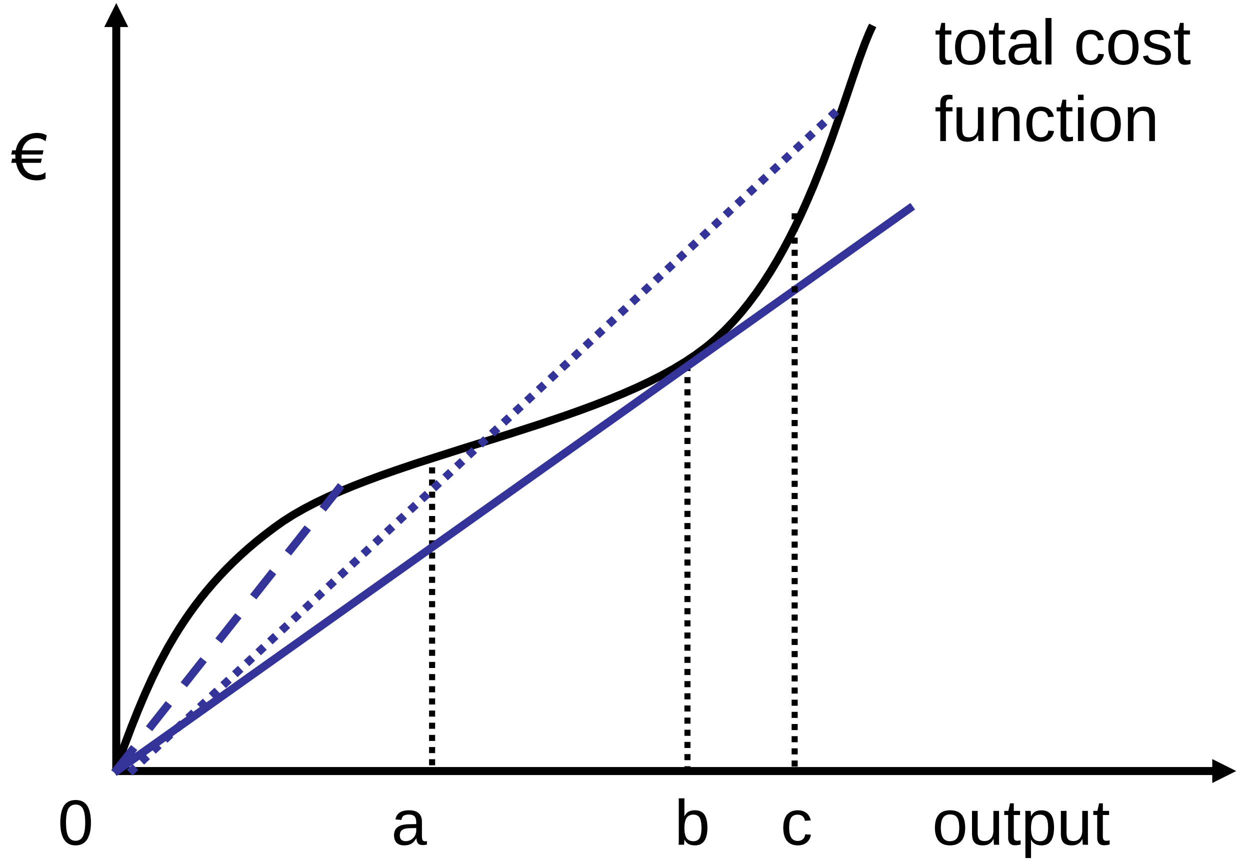


AC falls when $0 < q < b$.

AC rises when 

AC has a minimum at

At the minimum AC = MC. The chord is

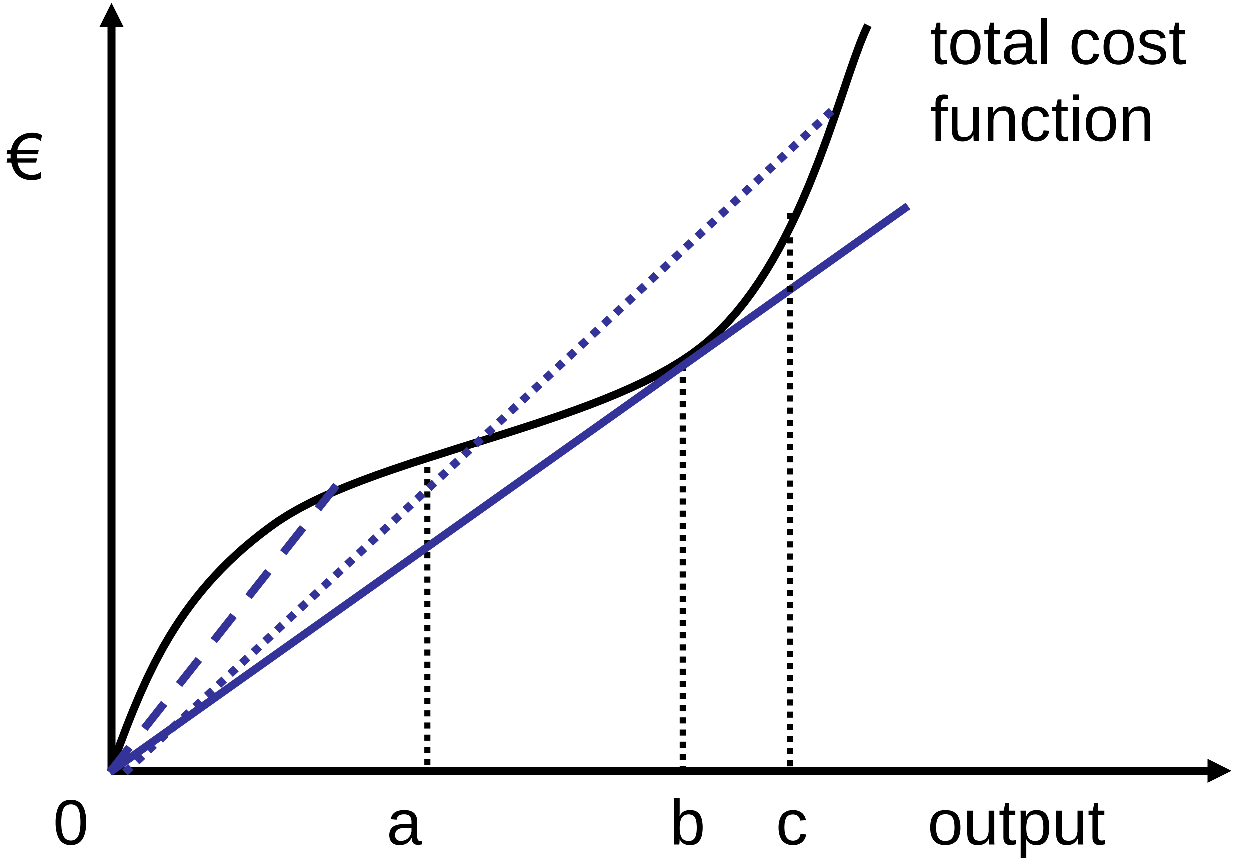


AC falls when $0 < q < b$.

AC rises when $b < q$.

AC has a minimum at 

At the minimum AC = MC. The chord is

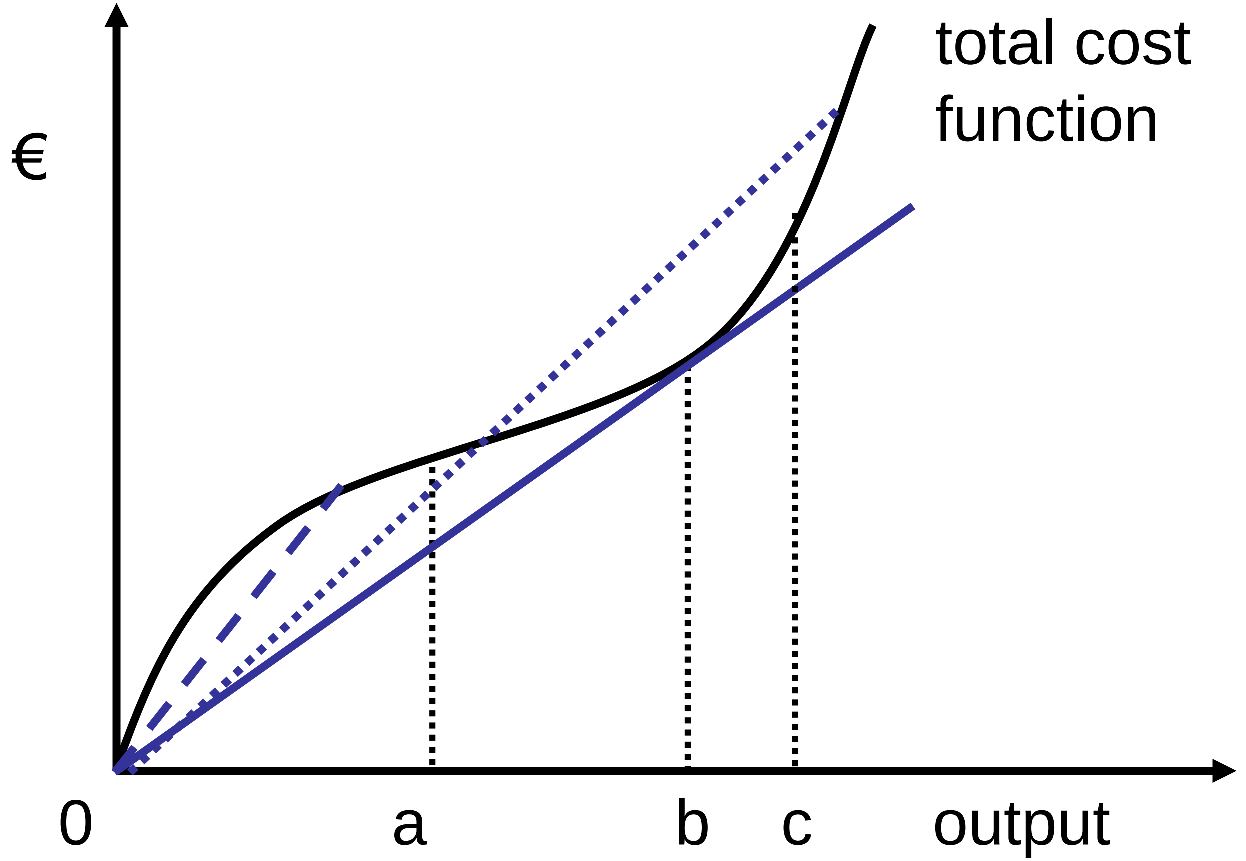


AC falls when $0 < q < b$.

AC rises when $b < q$.

AC has a minimum at b.

At the minimum AC = MC. The chord is 

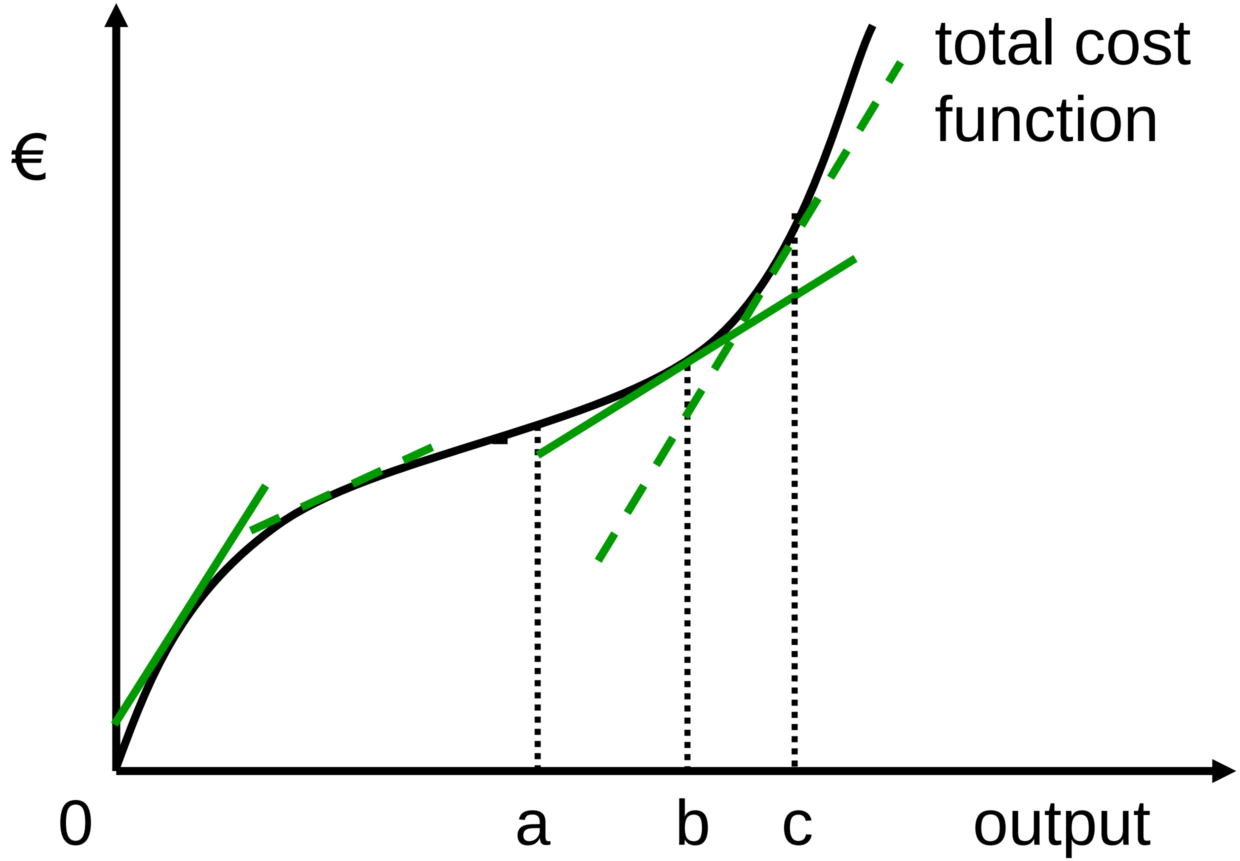


AC falls when 0 < q < b.

AC rises when b < q.

AC has a minimum at b.

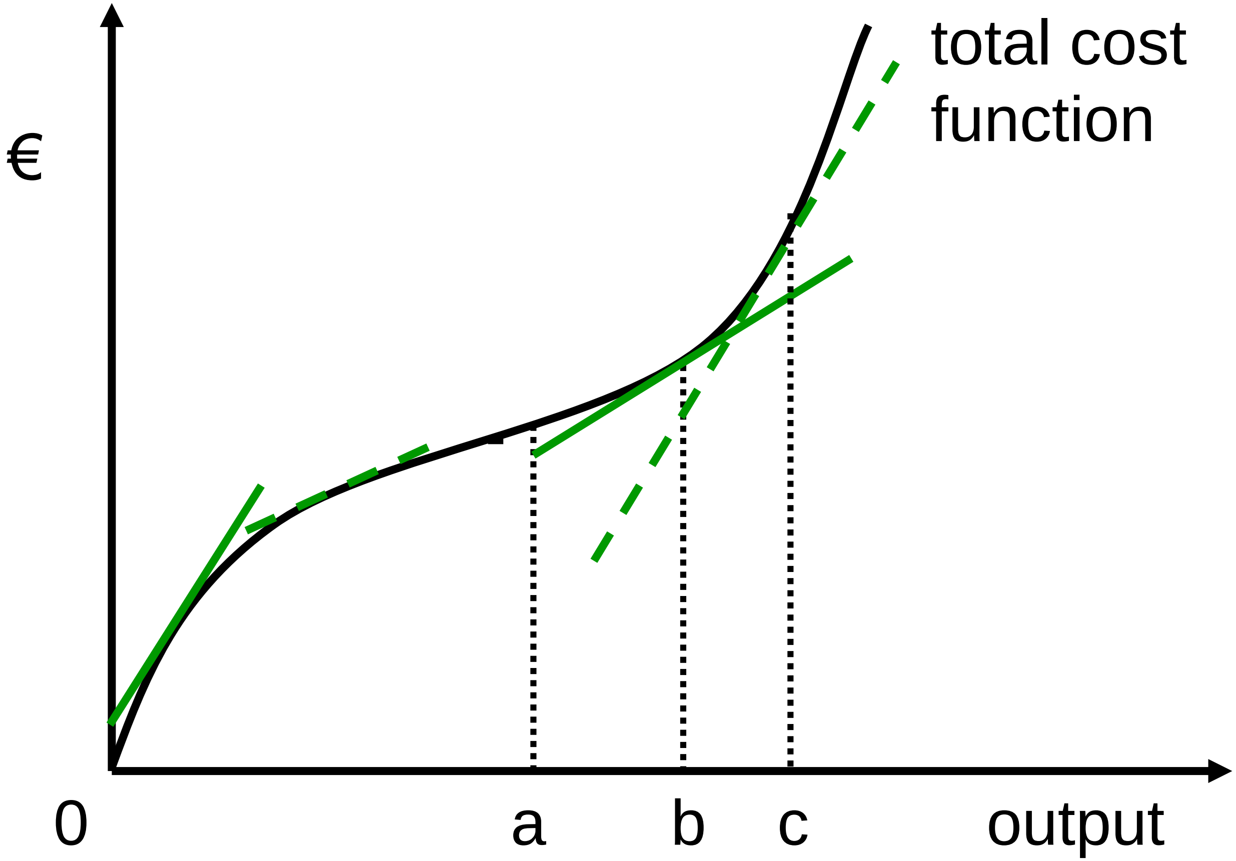
At the minimum AC = MC.   The chord is also a tangent.



MC falls when 

MC rises when

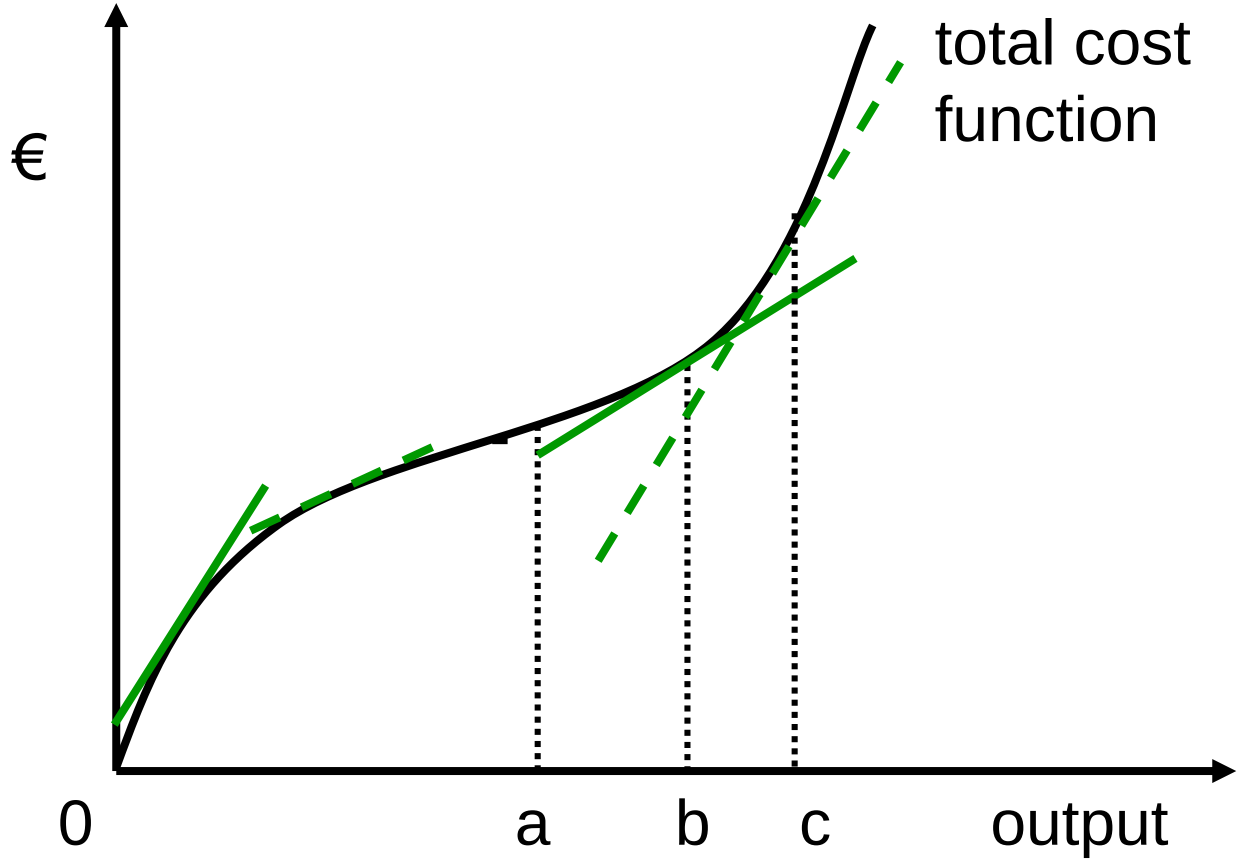
MC has a minimum when



MC falls when 0 < q < a.

MC rises when

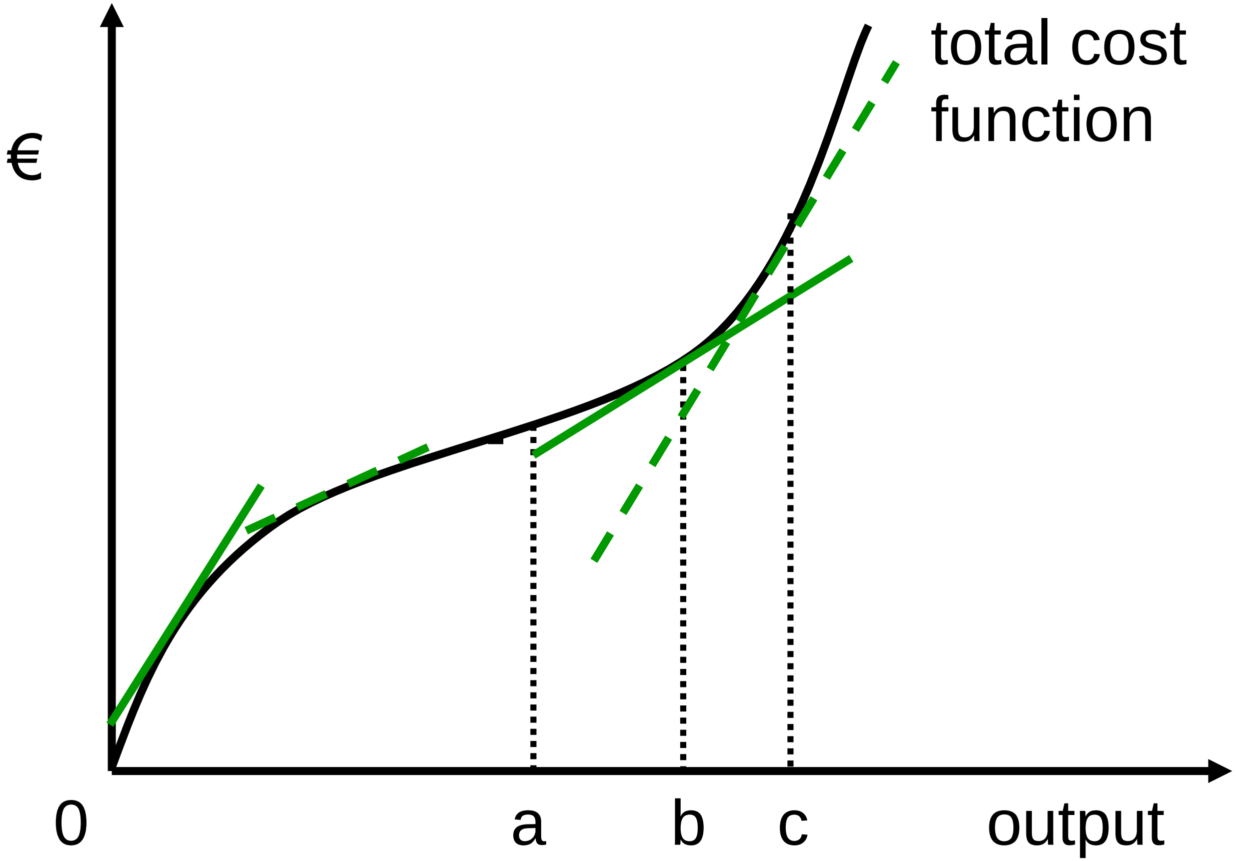
MC has a minimum when



MC falls when $0 < q < a$.

MC rises when $a < q$.

MC has a minimum when 

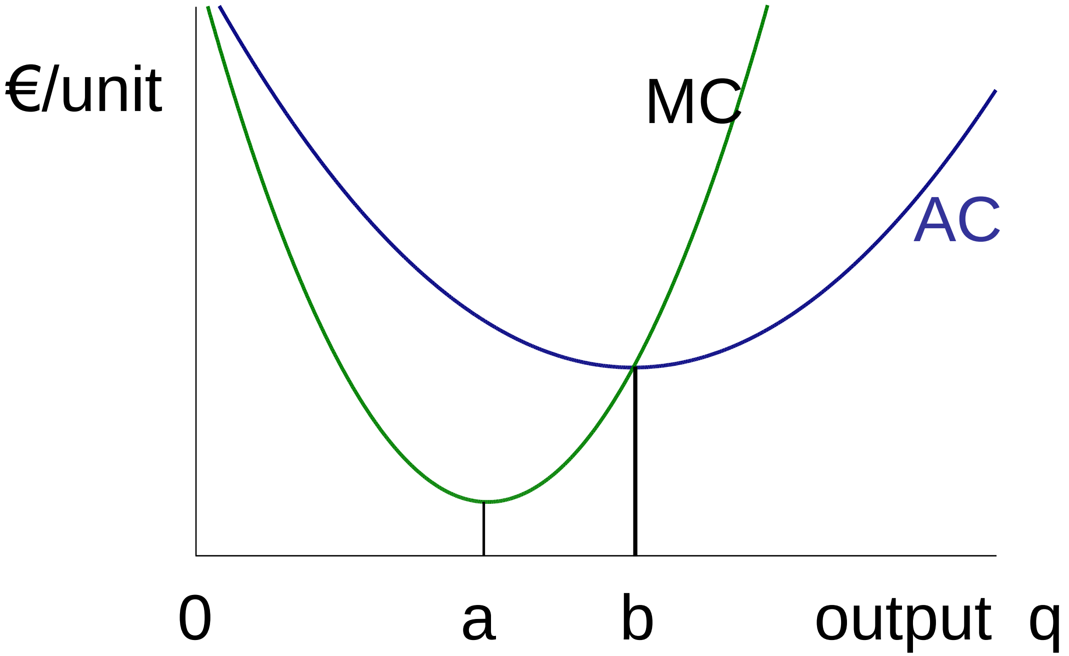


MC falls when 0 < q < a.

MC rises when a < q.

MC has a minimum when q = a.

Marginal and average cost with a u shaped average cost curve

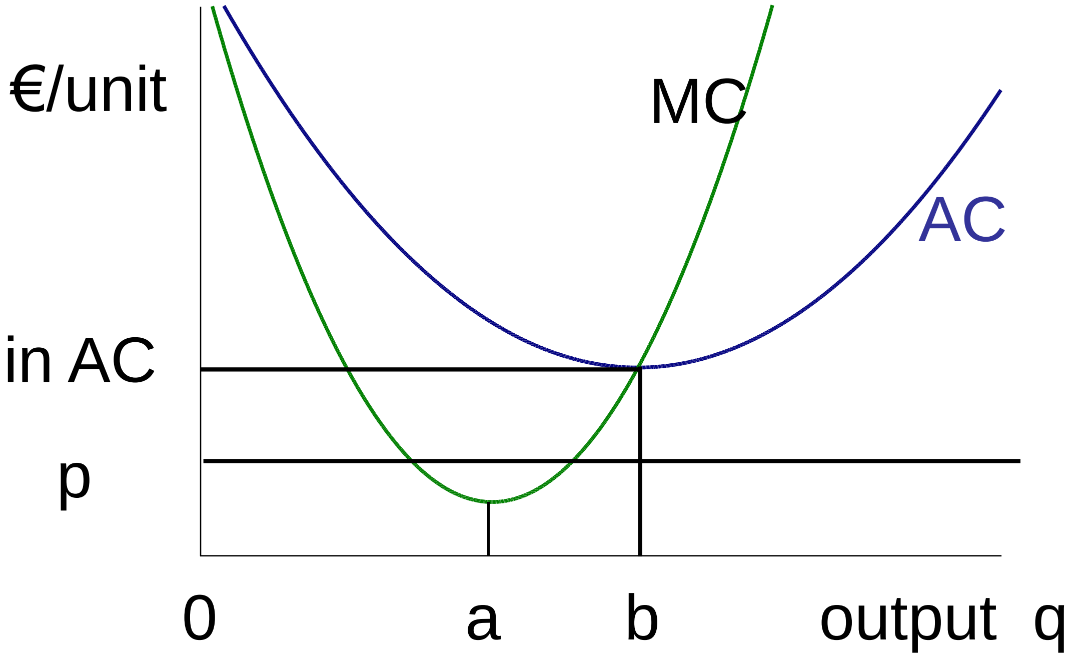


MC deceases when q < a and increases when q > a.

AC decreases when q < b and decreases when q > b.

At b AC has a minimum so its derivative is 0 implying that MC = AC.

Profit maximization by a price taking firm with a u average shaped cost curve

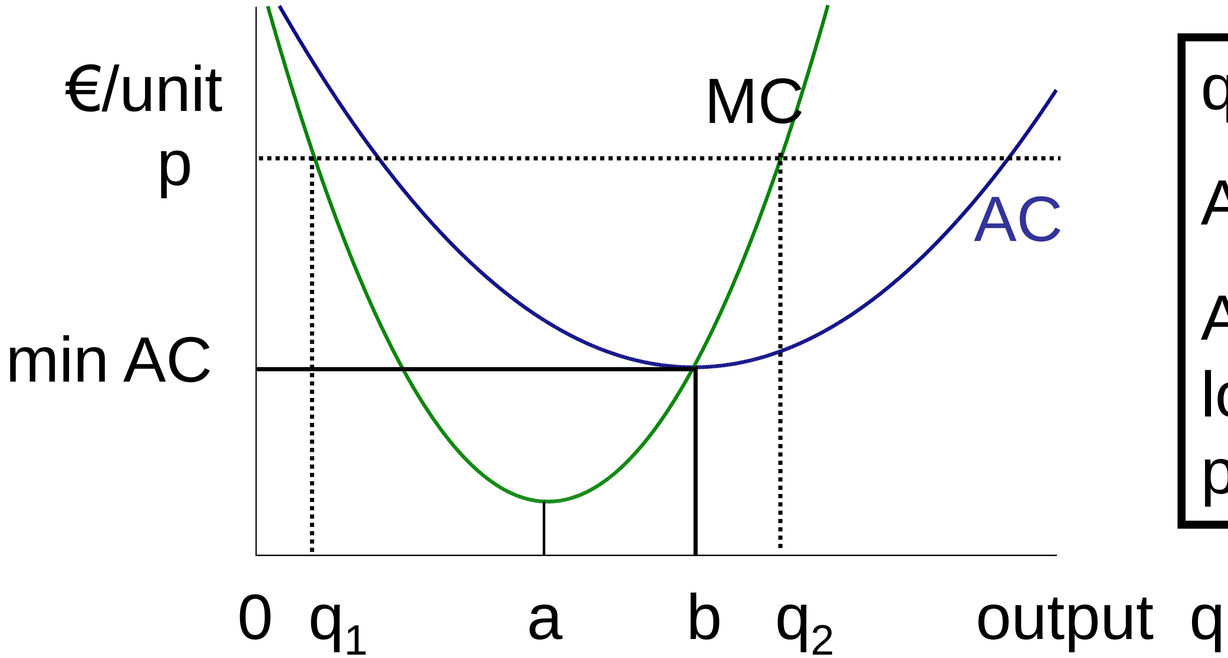


Here p < min AC

the firm makes losses at all q > 0,

the shut down condition cannot be satisfied at q > 0 so q = 0.

Profit maximization by a price taking firm with a u average shaped cost curve



q_2 maximizes profits.

At q_2 p = MC.

At q_1 p = MC, this is a local minimum of profits.

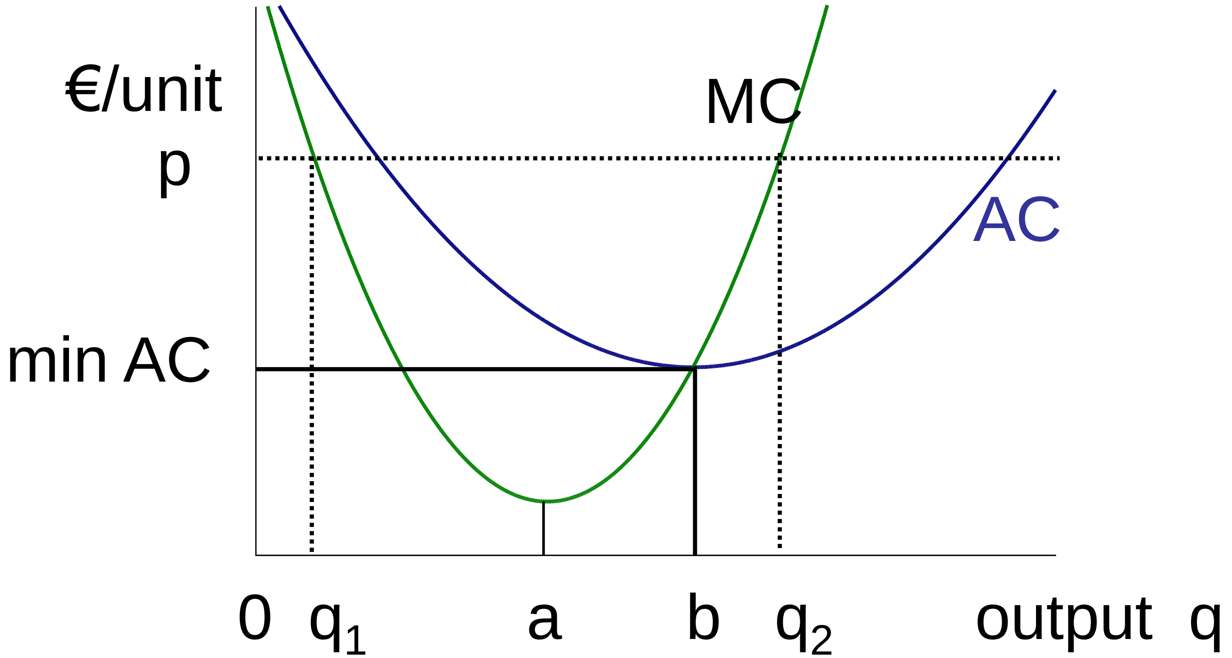
Here p > min AC. The firm can make profits.

When $q < q_1$ $p < MC$ so increasing q decreases profits.

When $q_1 < q < q_2$ $p > MC$ increasing q increases profits.

When $q_2 < q$ $p < MC$ increasing q decreases profits.

Profit maximization by a price taking firm with a u average shaped cost curve

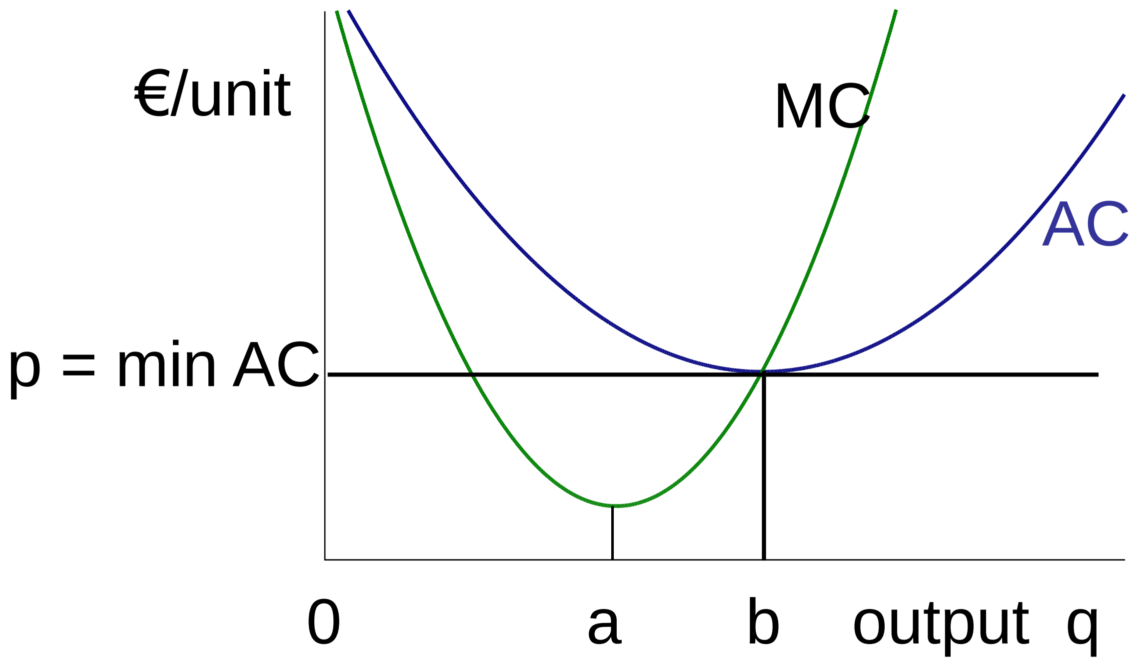


At q_2 p = MC > AC so the firm makes profits > 0. Shutdown & output conditions are satisfied. q_2 maximizes profits

Price = MC at q_1 but q_1 does not maximize profits.

q_1 gives a profit minimum.

Profit maximization by a price taking firm with a u average shaped cost curve



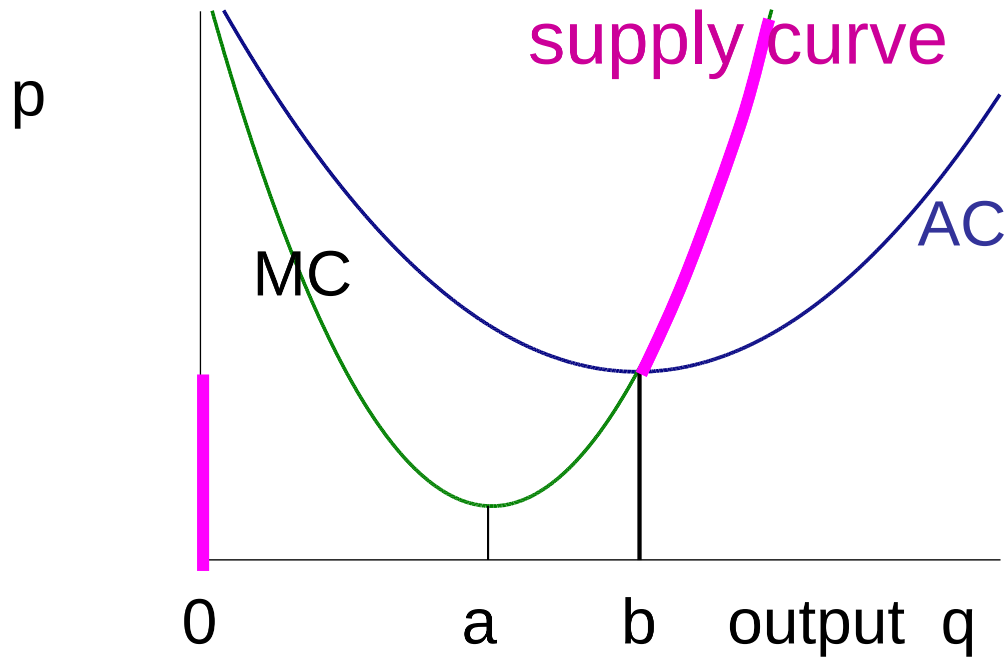
Here p = min AC. The firm can make at most 0 profits.

It does this by producing b or producing 0.

At the output b that minimizes AC p = MC = AC.

Both the shut down and output conditions are satisfied.

Profit maximization by a price taking firm with a u average shaped cost curve



The supply curve is the upward sloping part of the MC curve where MC ≥ AC.

When p < min AC the firm produces 0.

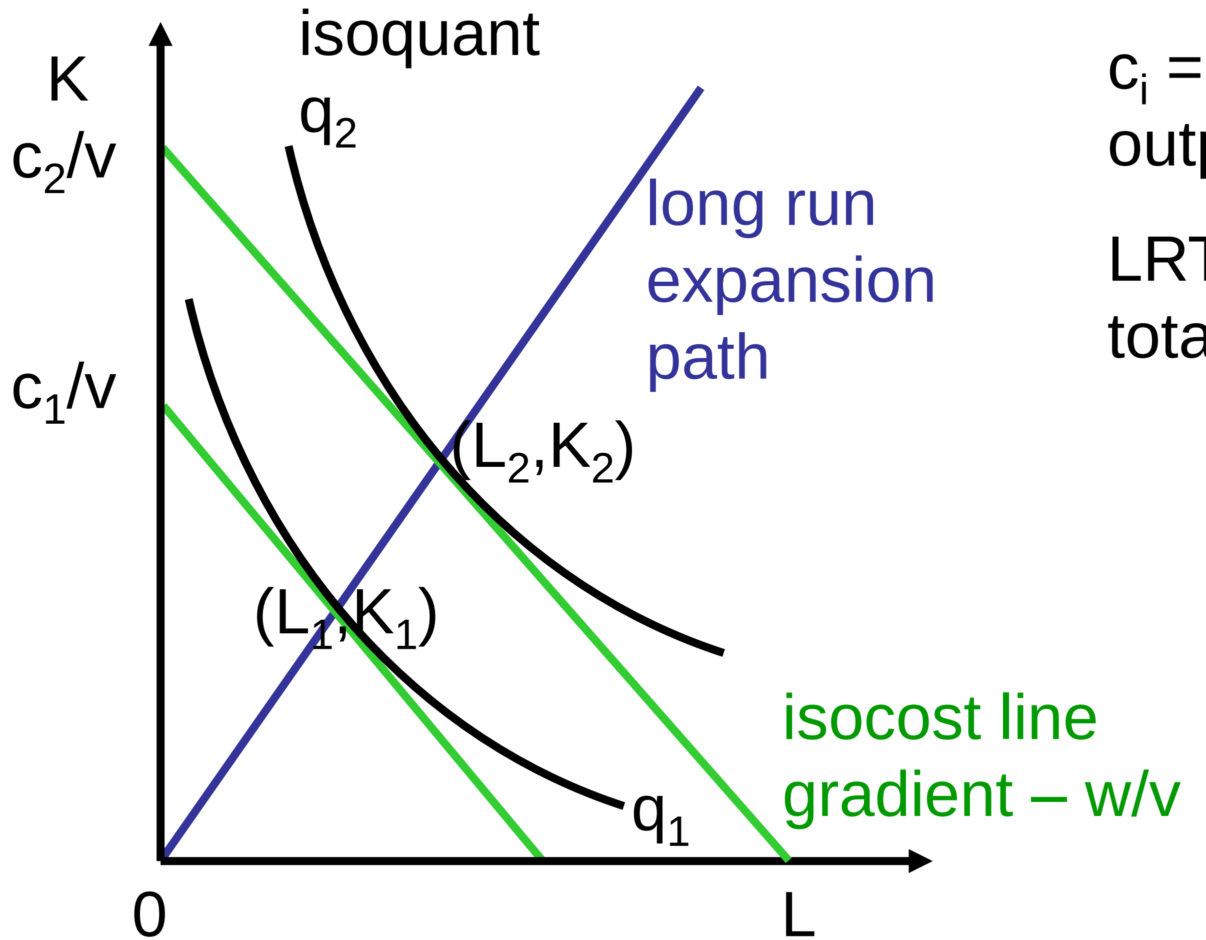
When p = min AC the firm produces either 0 or b.

When p > min AC the firm produces on the point where p = MC, MC is increasing and MC > AC.

7. Long run and short run costs and supply

7. Long run and short run costs and supply

- Up to now, all inputs are chosen at same time.
- Now there are two periods, the planning period and the production period.
- Capital is fixed in the planning period and paid for in the production period.
- Labour is chosen and paid for in the production period.
- If the firm knows output and input prices in the planning period this makes no difference.
- K and L are chosen to minimize total cost c(v,w,q).



c_i = LRTC of output q_i

LRTC = long run total cost

If in the planning period the firm knows output q and input prices w & v in the production period it chooses the cost minimizing point on the long run expansion path.

Total inputs (L_i,K_i) total cost LRTC = c_i when output is q_i.

Short Run Costs

- The firm installs K in the planning period, when it is uncertain what input prices will be and how much it will produce in the production period.
- The cost s(v,w,K,q) of production depends on input prices v and w, capital K and output q. It is called short run total cost (SRTC).
- Contrast LRTC c(v,w,q) does not depend on K.

Which menu is better?

1. Main: curry or steak

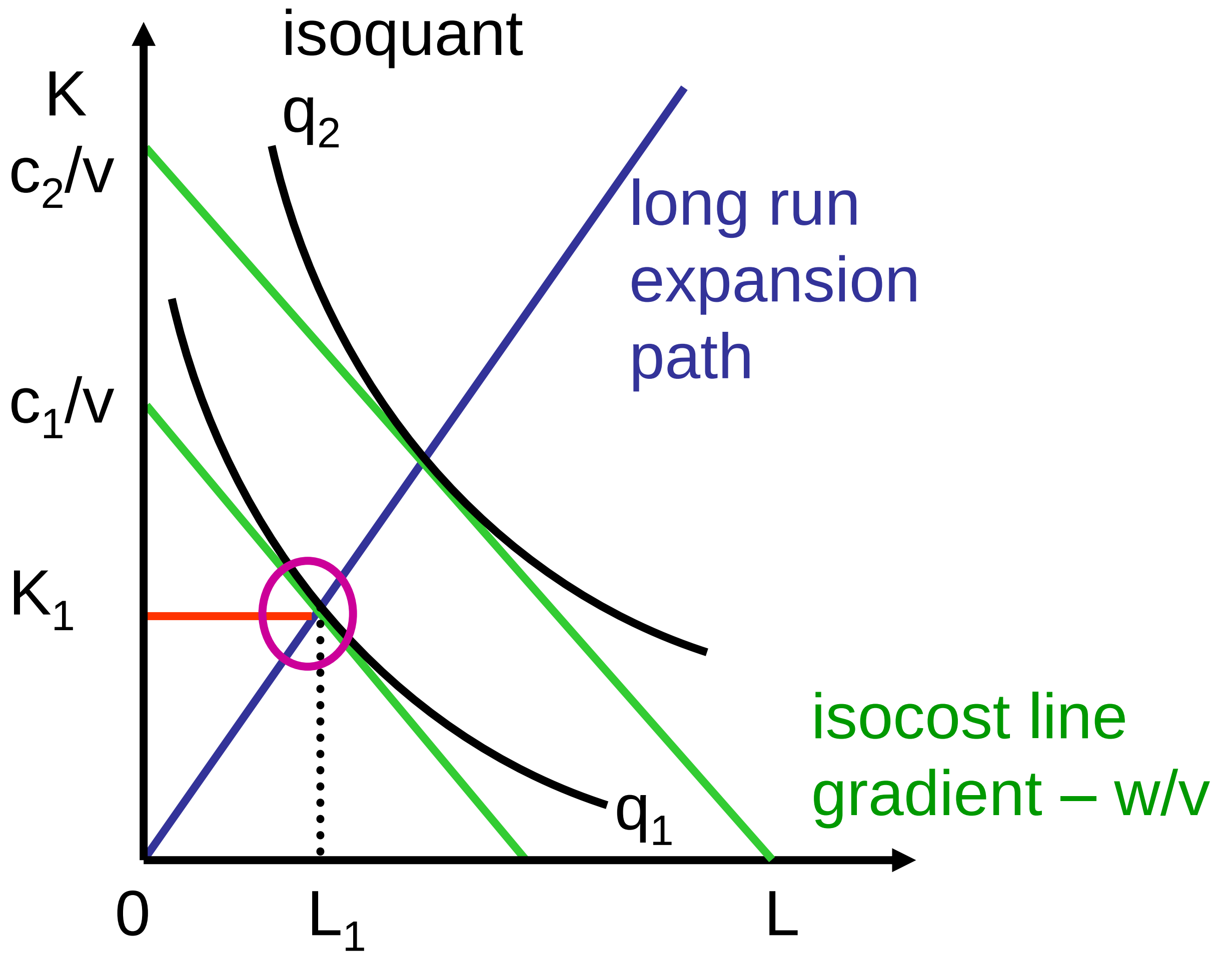
   dessert: fruit or cake

2. Main: curry or steak

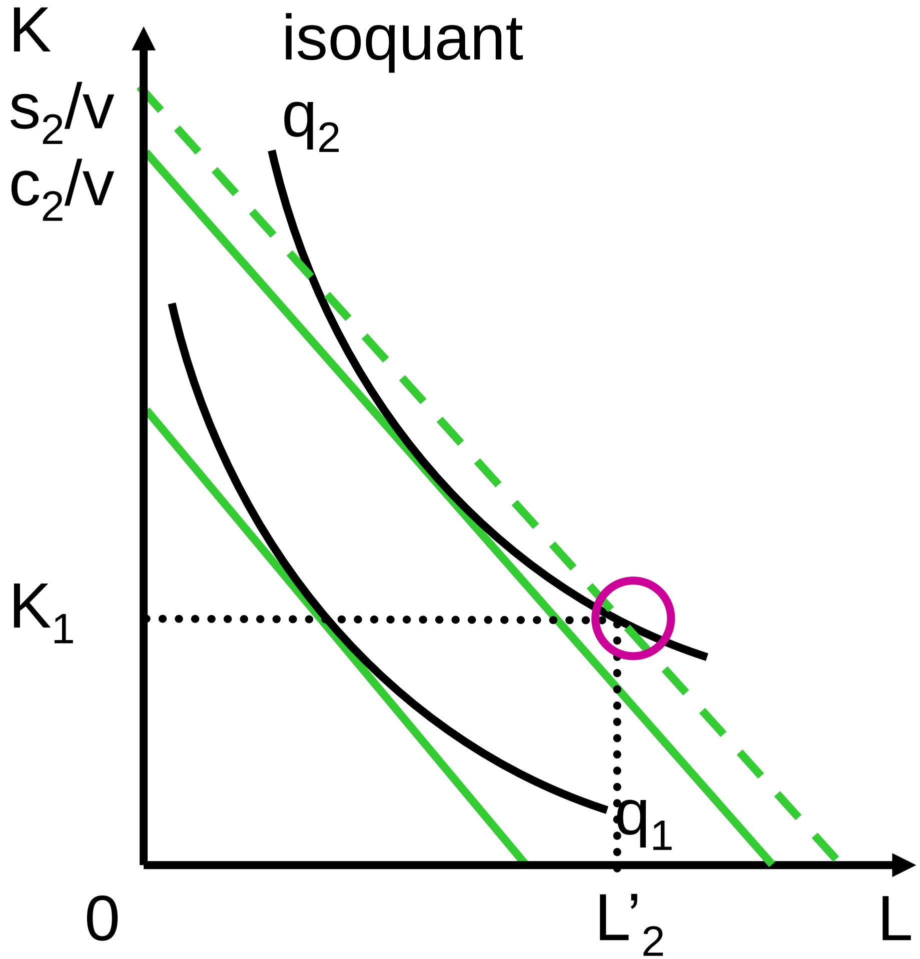
   dessert: fruit

Flexibility can never be bad in standard economics models

- If given the choice between fruit & cake you chose cake the more restricted menu is worse.
- If given the choice between fruit & cake you chose fruit the more restricted menu is no better.
- But limiting flexibility may be good if you know you will eat but regret the cake.
- Limiting flexibility can have advantages in games.



If in production period $K = K_1$, input prices are w, v and output is q_1 the minimum cost of producing q_1 is c_1.



c_2 = LRTC of output q_2

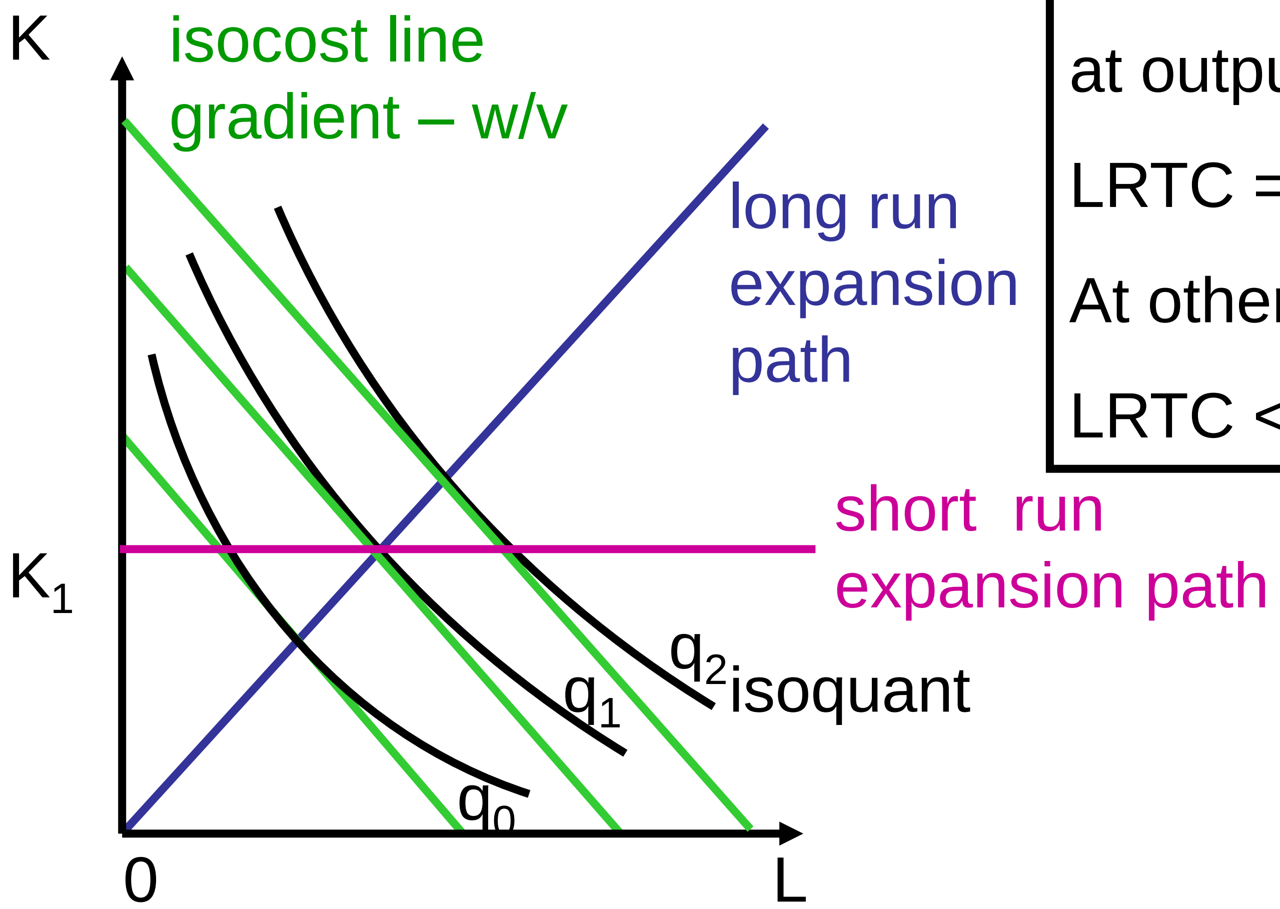
s_2 = SRTC of output q_2

SRTC = short run total cost

$s_2 > c_2$

If in production period $K = K_1$, input prices are w, v and output is q_2 the firm has to use L_2' labour.

Total cost $s_2/v > c_2/v$.



With capital K_1

at output q_1

LRTC = SRTC.

At other outputs

LRTC < SRTC.

With capital fixed at K_1 in the planning period the firm is on the short run expansion path in the production period.

At output q_1 the firm is on the long run expansion path.

At other outputs the firm is not on the long run expansion path.

LRTC ≤ SRTC always

- With a minimization problem you can often do better and never do worse with more flexibility.

- Long run (choose K and L)

  more flexible than

  short run (choose L, K fixed).

So either LRTC < SRTC or LRTC = SRTC

Long run and short run costs and supply with a Cobb-Douglas production function

Finding SRTC with a Cobb-Douglas Production Function

Rearrange the production function equation

$q = K^{3/5} L^{2/5}$ to get $L = q^{5/2} K^{-3/2}$

So to produce q units of output with K = K* requires

$q^{5/2} K^{*-3/2}$ units of labour and at prices v and w costs

$SRTC = w\, q^{5/2} K^{*-3/2} + vK^*$.

I have already found that when K and L are chosen freely

$LRTC = [(3/2)^{2/5} + (2/3)^{3/5}]\, w^{2/5}\, v^{3/5}\, q$

LRTC depend on v,w,q

SRTC depend on v,w,K*,q

Total, average and marginal long run costs

Long run total cost	LRTC	$c(v,w,q)$
Long run average cost	LRAC	$\dfrac{c(v,w,q)}{q}$
Long run marginal cost	LRMC	$\dfrac{\partial c(v,w,q)}{\partial q}$

Total, average and marginal short run costs

Short run total cost	SRTC	$s(v, w, K^*, q)$
Short run average cost	SRAC	$\dfrac{s(v, w, K^*, q)}{q}$
Short run marginal cost	SRMC	$\dfrac{\partial s(v, w, K^*, q)}{\partial q}$

With the Cobb-Douglas production function $q = K^{3/5}L^{2/5}$

$LRTC = [(3/2)^{2/5} + (2/3)^{3/5}]w^{2/5}v^{3/5}\, q$ so

$LRAC = LRMC = \quad [(3/2)^{2/5} + (2/3)^{3/5}]w^{2/5}v^{3/5}$

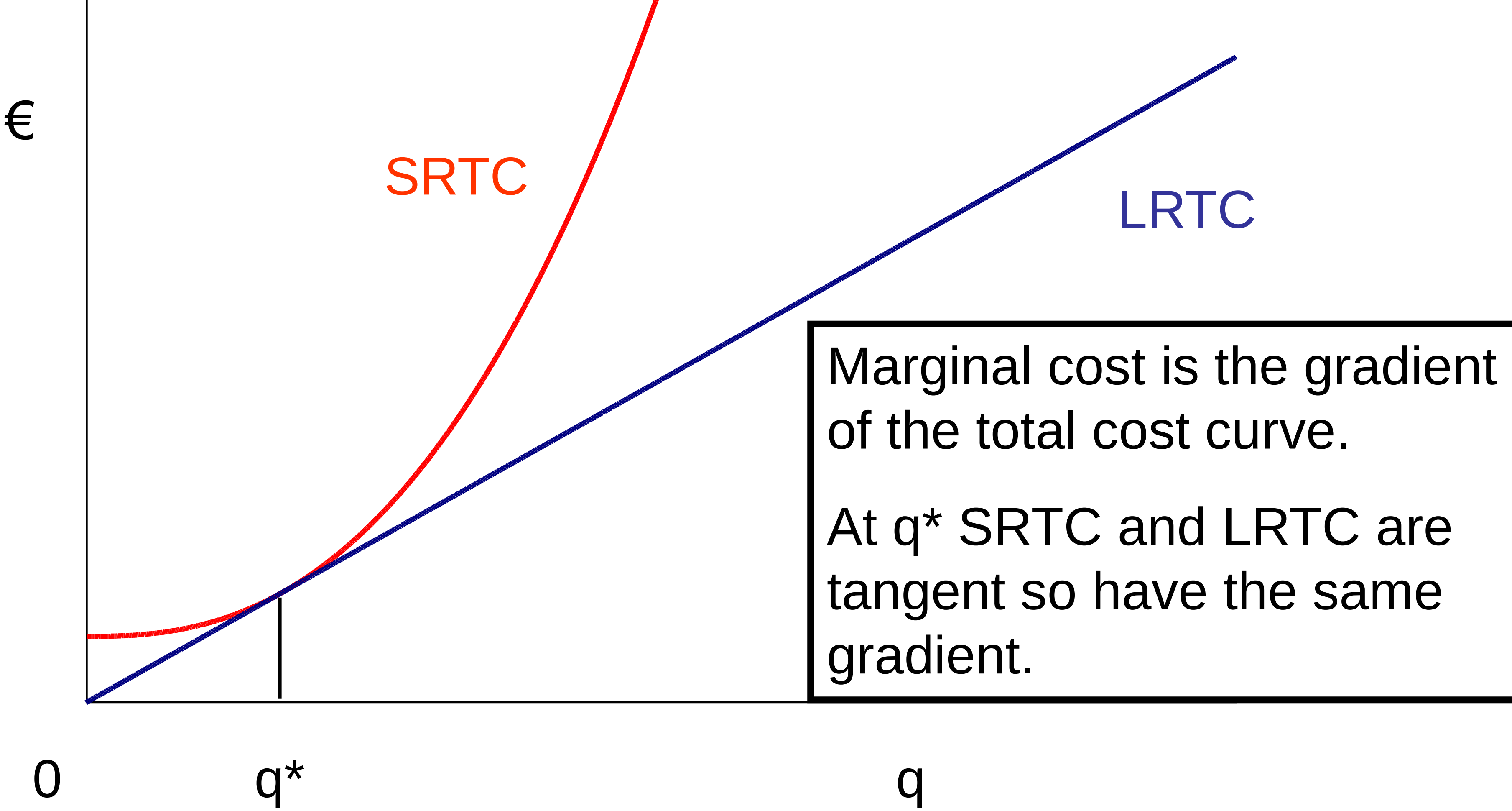
The production function has constant returns to scale, long run average and marginal costs are equal and do not depend on output.

$SRTC = w\, q^{5/2}\, K^{*\,-3/2} \;+\; vK^*$ so

$SRAC = \; w\, q^{3/2}\, K^{*\,-3/2} \;+\; vK^* q^{-1}$

$SRMC = w(5/2)\; q^{3/2}\, K^{*\,-3/2}$

Short run average and marginal costs depend on output.



Marginal cost is the gradient of the total cost curve.

At q* SRTC and LRTC are tangent so have the same gradient.

At q* the fixed level of capital K* is at the cost minimizing level and LRTC = SRTC so LRAC = SRAC.

As SRTC ≥ LRTC for all values of q, SRAC ≥ LRAC for all q.

LRTC and SRTC are tangent at q* where LRMC = SRMC.

With the Cobb-Douglas production function $q = K^{3/5}L^{2/5}$

$$LRTC = [(3/2)^{2/5} + (2/3)^{3/5}]w^{2/5}v^{3/5}\, q \quad \text{so}$$

$$LRAC = LRMC = [(3/2)^{2/5} + (2/3)^{3/5}]w^{2/5}v^{3/5}$$

The production function has constant returns to scale, long run average and marginal costs are equal and do not depend on output.

$$SRTC = w\, q^{5/2}\, K^{*\,-3/2} + vK^* \quad \text{so}$$

$$SRAC = w\, q^{3/2}\, K^{*\,-3/2} + vK^*q^{-1}$$

$$SRMC = w(5/2)\, q^{3/2}\, K^{*\,-3/2}$$

Short run average and marginal costs depend on output.

$$LRAC = LRMC = \quad [(3/2)^{2/5} + (2/3)^{3/5}]w^{2/5}\mathrm{v}^{3/5}$$

does not vary with q so graph is a horizontal straight line

$SRMC = w(5/2)\ q^{3/2}\ K^{*-3/2}$ has derivative with respect to q

$$(15/4)wq^{1/2}\ K^{*-3/2} > 0$$

so is increasing.

$$SRAC = w\, q^{3/2}\, K^{*-3/2} + vK^{*}q^{-1}$$

$$SRMC = w(5/2)\, q^{3/2}\, K^{*-3/2}$$

so $SRMC - SRAC$

$$= q^{3/2}\, K^{*-3/2} w\left(\frac{5}{2} - 1\right) - vK^{*}q^{-1}$$

$$= \frac{3}{2} q^{3/2}\, wK^{*-3/2} - vK^{*}q^{-1}$$

so $SRMC \geq SRAC$ if

$$\frac{3}{2} q^{3/2} w\, K^{*\,-3/2} \geq vK^* q^{-1} \text{ or equivalently } \frac{3}{2} w\, q^{5/2}\, K^{*\,-3/2} \;\geq\; vK^*$$

which implies that $q^{5/2} \;\geq\; \dfrac{2v}{3w} K^{*\,5/2}$

or $q \geq q^*$ where $q^* = \left(\dfrac{2v}{3w}\right)^{2/5} K^*$

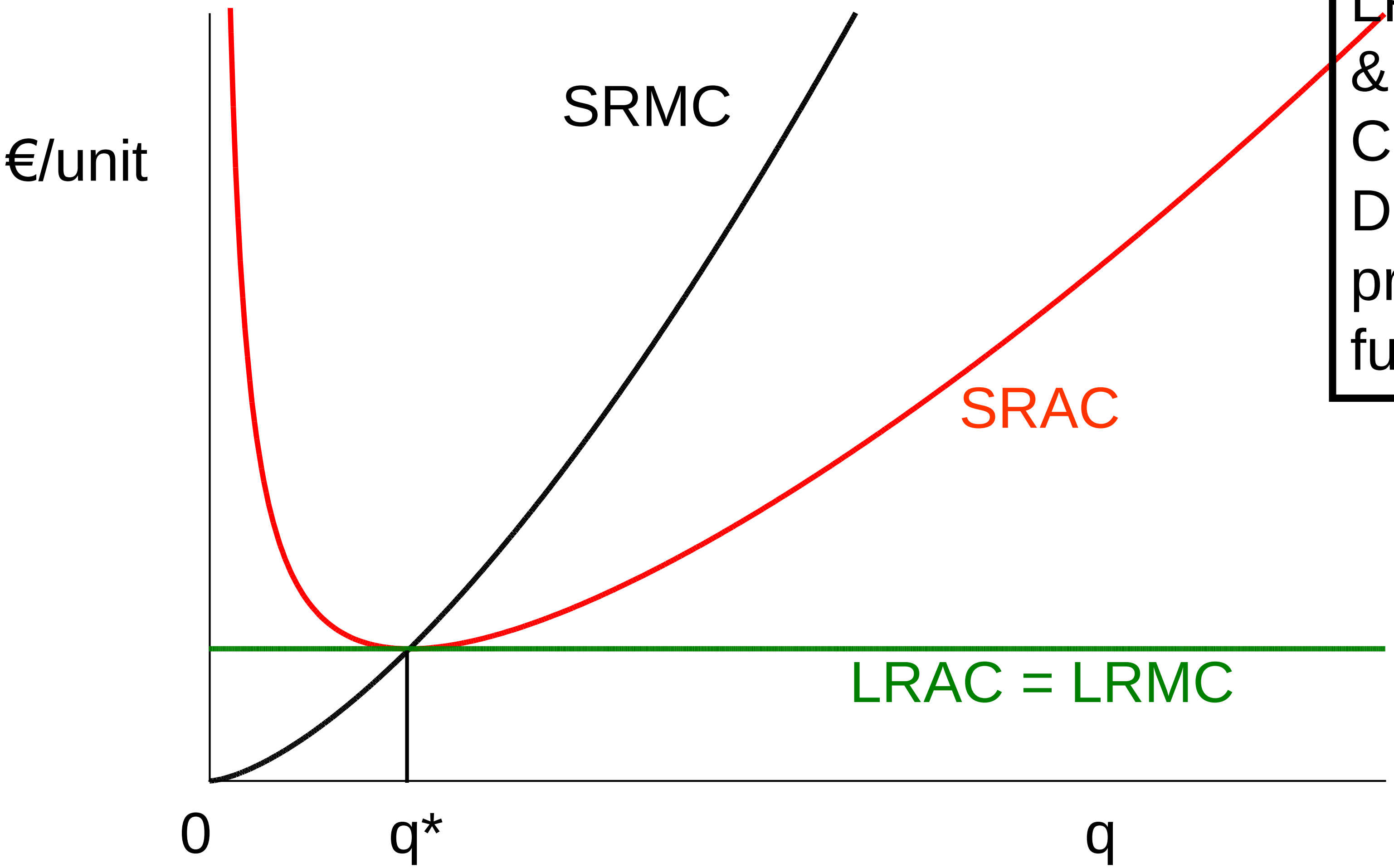
so $SRAC$ is decreasing when $q \leq q^*$

increasing when $q \geq q^*$

SRAC is u-shaped
with a minimum at q*

With some calculus, you can establish that

- LRMC = LRAC is a horizontal straight line.
- SRMC is 0 at q = 0 and increasing
- SRAC is u-shaped
  - (do this by seeing where SRMC > SRAC)
- SRAC tends to infinity as q tends to 0
- SRAC = SRMC= LRMC = LRAC at the point that minimizes SRAC



LR & SR MC & AC with this Cobb-Douglas production function.

At q = q* LRAC = LRMC = SRAC = SRMC

At all values of q   SRAC ≥ LRAC

SRMC can be > or < LRMC

The General Relationship Between SR & LR
AC & MC

The General Relationship Between SR & LR AC & MC

SRAC ≥ LRAC for all q

SRAC = LRAC and SRMC = LRMC at q* where the capital stock K* would be chosen in the long run.

In the next slides the firm plans for q*.

Perfect competition

Does not imply

LRAC = SRAC = LRMC = SRMC

Why?

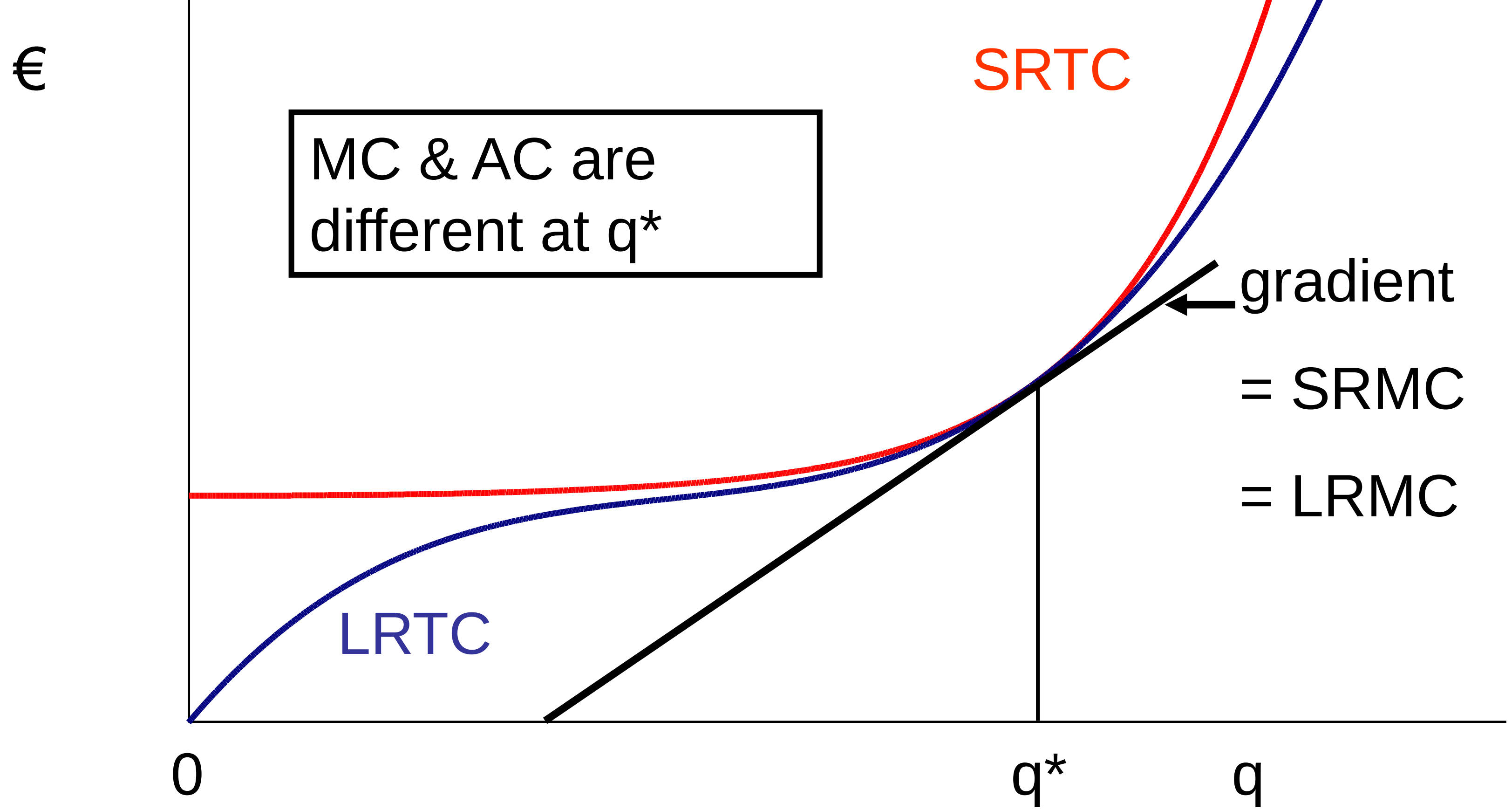
The next 2 slides

show a situation in which the capital stock is at the level that minimizes the long run cost of producing q^*, but q^* is **not at** the level of output that minimizes LRAC.

In this case at q^* LRAC = SRAC

& LRMC = SRMC.

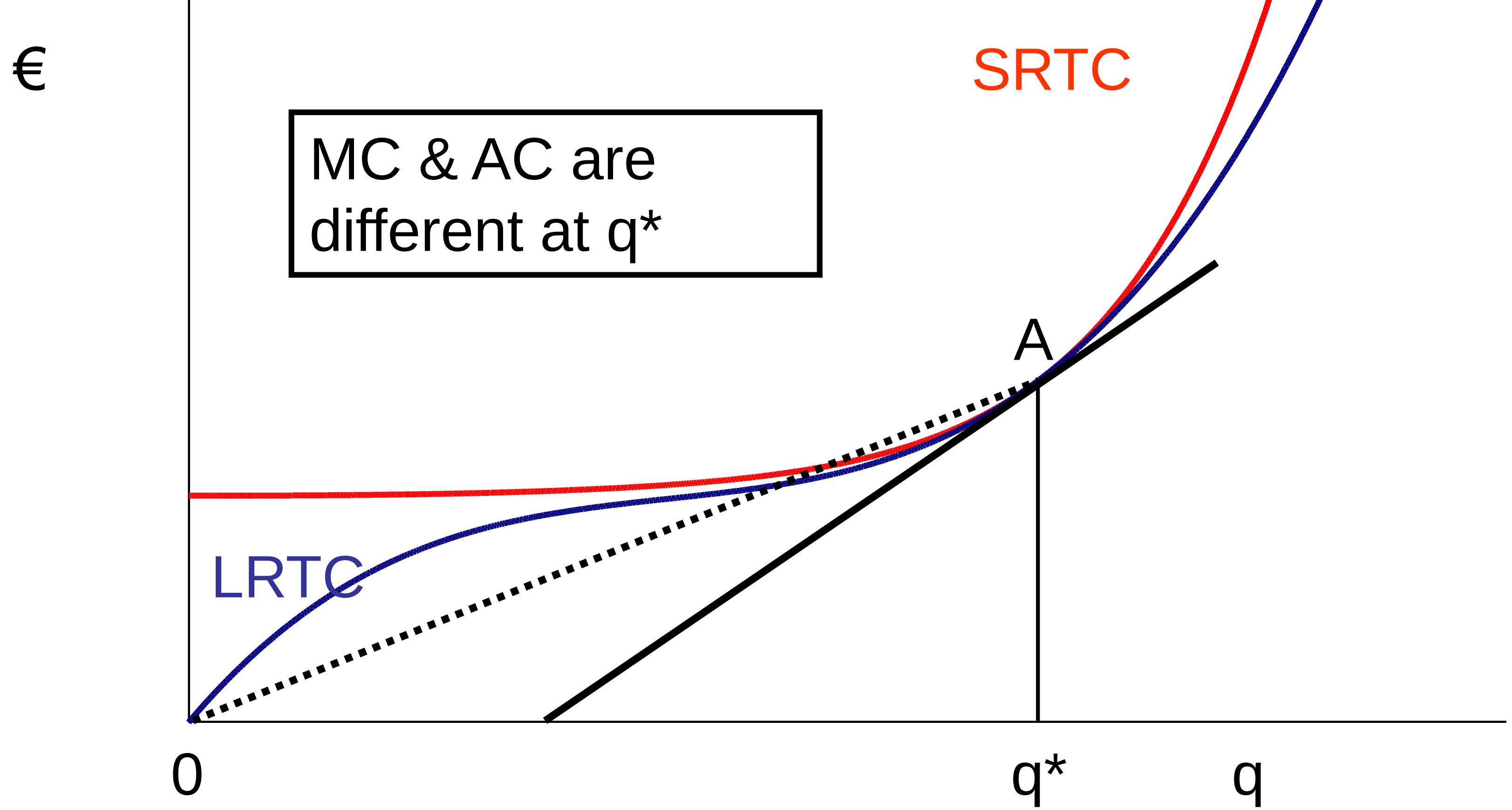
But LRAC = SRAC $\neq$ LRMC = SRMC



Here at q* the fixed level of capital K* is at the cost minimizing level so LRTC = SRTC so LRAC = SRAC.

As SRTC ≥ LRTC for all values of q, SRAC ≥ LRAC for all q.

At q* LRTC and SRTC curves are tangent so the gradient MC is the same.



At q* LRAC = SRAC and LRMC = SRMC

But average & marginal costs are different

MC = gradient tangent

AC = gradient chord 0A

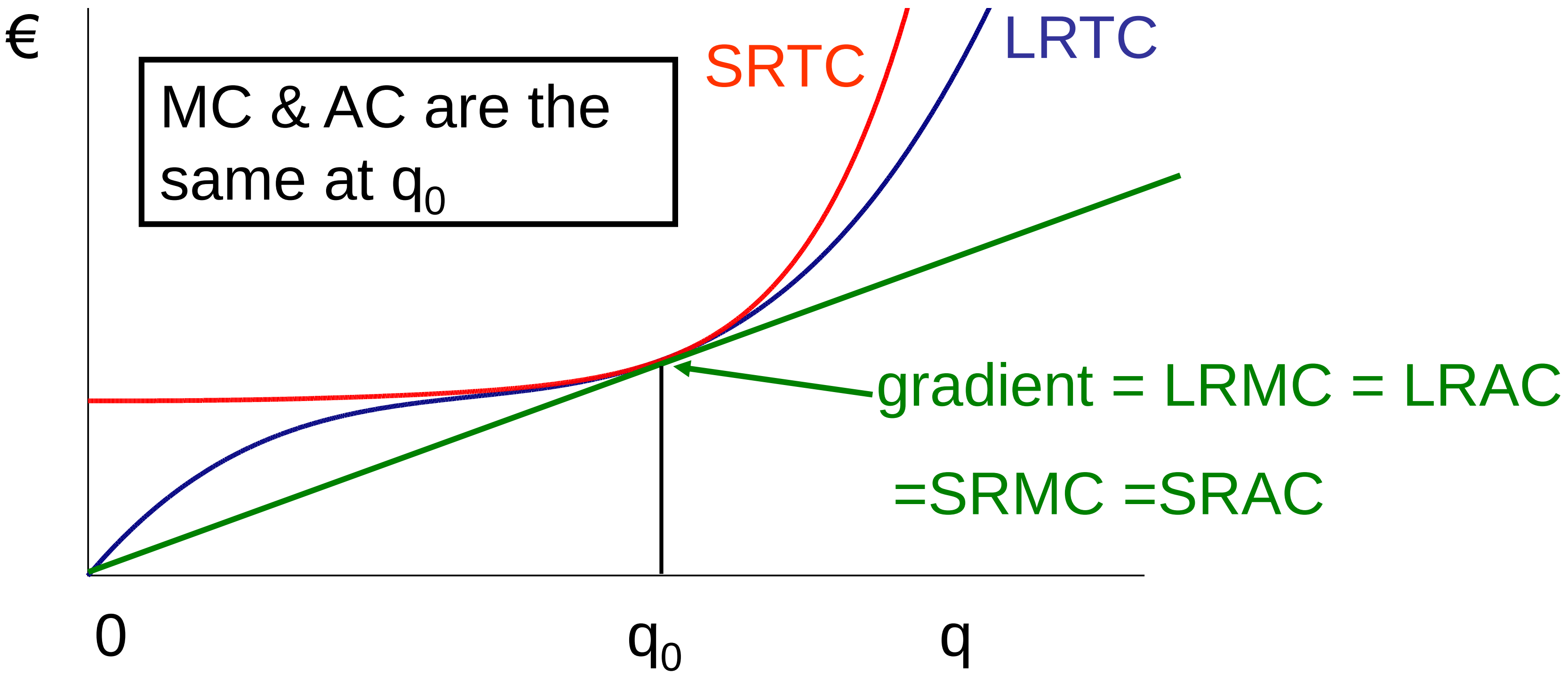
The next slide

show a situation in which the capital stock is at the level the minimizes the long run cost of producing q_0

where q_0 is the level of output that minimizes LRAC.

In this case only LRAC = SRAC = LRMC = SRMC at q_0.

LRS SRS with U shaped AC



Suppose the firm plans to produce at the level q_0 of q that minimizes LRAC.   At this level LRAC = LRMC.

It installs the cost minimizing level of K for q_0 so at q_0 SRAC = LRAC and SRMC = LRMC.

Is the short run long run analysis useful?

What does long run and short run mean?

Economics textbooks:

in the long run all inputs are variable

in the short run some inputs are fixed.

Everyday sense

short run: a temporary state that won’t persist

long run: the state things tend to revert to.

The meanings are similar if the economy tends to come back to a steady state. Does it?

Is the short run long run analysis useful?

Yes, it captures the idea that unexpected things happen, and if a firm has to decide on some inputs in advance it may maximize economic profits by producing even if it makes an economic loss.

But it is a simple model, and can be very misleading.

What has been achieved

- The implications for supply of profit maximization given production and cost functions.
- No discussion of the firm as an organization.
- No product differentiation.